Modelling of irrigation channel dynamics for controller design

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ABSTRACT

Irrigation canals are complex hydraulic systems difficult to control. Design methods have been developed using linear control theory. To use this tool, a linear model of the dynamic is needed.This paper presents a simple model for canal reach dynamics. A reach transfer matrix is obtained by linearisation of Saint Venant equations near a steady flow regime.The accuracy of this transfer matrix is evaluated, in frequency and time domain.

1. INTRODUCTION

The hydraulic behaviour of irrigation canals shows that these systems are complex, with a dynamic characterised by important time lags, strong nonlinearity and numerous interactions between different consecutive sub-systems. A good knowledge of system dynamics is needed to design an automatic controller for an irrigation canal.

It is possible to split a canal into sub-systems composed of a reach with a cross regulator at is downstream end. Cross structure dynamic, for example a gate, is simple to model. On the other hand reach dynamics is modelled by a set of partial derivative hyperbolic equations: Saint-Venant's equations.

Linearisation of Saint-Venant's equations near a steady flow, allows to obtain a reach transfer matrix that has the advantage to keep the distributed parameter system characteristics and therefore the infinite state space dimension.

The accuracy of this transfer matrix is evaluated for two kinds of reach behaviour : a short reach with waves propagation, a long reach with delay and damped wave motion. This evaluation is made by using a model based on a finite difference approximation of Saint-Venant's equations by Preissmann's scheme.

2. REACH DYNAMICS

Fluctuations in discharge and water depth in a canal pool are due to two physical phenomena occurring in the flow: wave propagation (perturbation), mass transport (long waves). If gate movements are slow, water depth and discharge change slowly and the flow is unsteady gradually varied. Assume that the flow is one-dimensional, streamline curvature is small and velocity is uniform over the cross section, the flow can be modelled very accurately by Saint Venant equations [1]. These equations are not valid to model cross structure behaviour. Cross structure equations are very numerous and are not valid for all kinds of flow (submerged, free flow...). The general form is: $Q = f(Z_u, Z_d, W)$ with Q the discharge, Z_u the upstream elevation, Z_d the downstream elevation and W the gate opening. For example for a submerged flow gate the equation is:

$Q = c_d\, L \sqrt{2g}\, W\, (Z_u - Z_d)^{1/2}$ with c_d the discharge coefficient and L the gate width.

The fluctuations in discharges and depths in a reach depend on many parameters such as reach length, bed slope, cross section, roughness, initial water profile. In order to characterise reach behaviour without studying the influence of each variable, the flow is characterised using dimensionless numbers. To transform Saint-Venant's equations in dimensionless form each variable is divided by a constant reference with the same dimension [2]. After simplifications, continuity and dynamic equations become:

$$\frac{\partial A^*}{\partial t^*} + \frac{\partial Q^*}{\partial x^*} = 0$$

$$\frac{\partial Q^*}{\partial t^*} + \frac{\partial Q^{*2}/A^*}{\partial x^*} + \frac{A^*}{F_r^2}\left(\frac{\partial Z^*}{\partial x^*} + \chi\, S_f^*\right) = 0$$

with A the cross section area, Q the discharge, Z the water elevation, S_f the friction slope, t the time, x the distance along the canal, g the gravity acceleration and * for dimensionless values.

If the reference condition is defined by the normal flow in a uniform reach with a bed slope S_b and a length X_r then:

$Fr_r^2 = \frac{Q_n^2 A_n}{g A_n^3 y_n} = \frac{Q_n^2 B_n}{g A_n^3}$ with $B_n = \frac{A_n}{y_n}$ so Fr can be interpreted as a Froude number.

$\chi = \frac{S_b X_r}{y_n}$ is a dimensionless length characteristic of the reach.

So the flow in a reach depends on two dimensionless numbers Fr and χ. The study done in [3] shows that χ characterises discharge propagation and $\eta = \frac{\chi}{Fr\,(1-Fr)}$ downstream level

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perturbations. For each dimensionless number, values are determined that characterise different kinds of behaviour.
The study of upstream to downstream discharge transfer function, for a wide rectangular channel, shows that 3 classes can be built. If $\chi < 3/5$ a first order is able to model the discharge dynamic. For $3/5 < \chi < 27/20$ a second order is needed and for $\chi > 27/20$ a second order with delay.
The study of downstream level to upstream level transfer function shows that if $\eta > 3$ there is no influence of the downstream perturbation on the upstream part of the reach, the wave is completely damped.
If the criteria for χ and η are crossed five kinds of behaviour can be found for a reach dynamic as χ and η are linked.
Principal characteristics are shown below:

	$0 < \eta < 3$	$\eta > 3$
$0 < \chi < 0.6$	type 1 Wave not damped flow modelled by a first order	type 3 Wave damped flow modelled by a first order
$0.6 < \chi < 1.35$	type 2 Wave not damped flow modelled by a second order	type 4 Wave damped flow modelled by a second order
$\chi > 1.35$		type 5 Wave damped flow modelled by a second order with delay

Table 1: Reach types

The whole study to determine the class was conducted for rectangular channels but it is supposed that conclusions are still true for different shapes.
Typical reach characteristics were found to represent each of the type described above. Reach is assumed uniform with a trapezoidal cross section and the discharge is assumed to vary from Q_M (design discharge) to $Q_m = Q_M/4$ keeping the same type of behaviour. That is to say χ and η stay in the same class. Two equal values are possible for η for two Froude numbers symmetric with respect to 0.5. Only the solution with Froude numbers less than 0.5, which corresponds to reaches with a mild slope more representative of the channels commonly regulated, are chosen. To give some examples, reach characteristics are shown here for type 1 and 5, where the subscript « M » is for maximum and « m » for minimum.

	Type 1	Type 5
η_m	1.038	6.578
η_M	2.432	14.339
χ_m	0.141	1.642
χ_M	0.308	3.535
B	7	8
m	1.5	1.5
S_b	0.0001	0.0008
X	3000	6000
K	50	50
Q_m	3.5	20
Q_M	14	80
y_m	0.97	1.36
y_M	2.12	2.92

Table 2: Reach characteristics

Where B is the bed width, m the side slope, y the uniform depth. K is the Strickler coefficient used to compute the friction slope.
These two reaches are used to test the transfer matrix.

3.TRANSFER MATRIX FOR THE REACH

To derive the reach transfer function of a reach some authors like Corriga [4] or Ermolin [5] linearize the Saint Venant equations near a steady state. They combine the dynamic and the continuity equation to get a differential equation of the second order in discharge (or in water depth). To solve the problem they used some hypothesis on the boundary conditions. By this way they get a transfer function for the reach with coefficient depending of these choices.
In this paper it is proposed to use also a linearisation of Saint Venant equations near a steady flow and to calculate the transfer matrix of the reach without any hypothesis on the boundary conditions. The result is the transfer matrix of the reach with its four components that modelize all the interactions in the reach between upstream and downstream discharges and levels.

Put: $A^* = A_0^* + A'$ and $Q^* = Q_0^* + Q'$
Linearisation of continuity equation gives :

$$\frac{\partial A'}{\partial t^*} + \frac{\partial Q'}{\partial x^*} = 0$$

That can be written using the water level:

$$k_s B_0^* \frac{\partial Z'}{\partial t^*} + \frac{\partial Q'}{\partial x^*} = 0 \qquad (1)$$

where k_s is a form coefficient for the cross section.
Dynamic equation is written as:

$$\frac{\partial Q'}{\partial t^*} + \frac{2Q_0^*}{A_0^*} \frac{\partial Q'}{\partial x^*} - \frac{2Q'Q_0^*}{A_0^{*2}} \frac{\partial A_0^*}{\partial x^*} - \left(\frac{k_s Q_0^{*2} B_0^*}{A_0^{*2}}\right) \frac{\partial Z'}{\partial x^*}$$
$$+ \left(\frac{2k_s Q_0^{*2} B_0^* Z'}{A_0^{*3}}\right) \frac{\partial A_0^*}{\partial x^*} + \frac{1}{F_r^2} \left(A_0^* \frac{\partial Z'}{\partial x^*} + k_s B_0 Z' \frac{\partial Z_0^*}{\partial x^*}\right)$$

$$+ \frac{\chi}{F_r^2}\,[k_s B_0^* Z'\, S_{f0}^* + A_0^*\, S_{f0}^* \,(\frac{2Q'}{Q_0^*} - \frac{10}{3}\frac{k_s B_0^* Z'}{A_0^*}$$

$$+\frac{4}{3}\frac{k_p \frac{\partial P_0}{\partial Z}}{P_0^*} Z')] = 0 \qquad (2)$$

Let's consider the Laplace transform of linearized Saint Venant equations (1) and (2):

$$k_s\, B_0^*\, s\, Z' + \frac{\partial Q'}{\partial x^*} = 0$$

$$sQ' + \frac{2Q_0^*}{A_0^*}\frac{\partial Q'}{\partial x^*} - \frac{2Q'Q_0^*}{A_0^{*2}}\frac{\partial A_0^*}{\partial x^*} - (\frac{k_s Q_0^{*2} B_0^*}{B_0^{*2}})\frac{\partial Z'}{\partial x^*}$$

$$+(\frac{2k_s Q_0^{*2} B_0^* Z'}{B_0^{*3}})\frac{\partial B_0^*}{\partial x^*} + \frac{1}{F_R^2}(B_0\frac{\partial Z'}{\partial x^*} + k_s B_0^* Z'\frac{\partial Z_0^*}{\partial x^*})$$

$$+ \frac{\chi}{F_r^2}\,[k_s B_0^* Z' S_{f0}^* + A_0^*\, S_{f0}^* \,(\frac{2Q'}{Q_0^*} - \frac{10}{3}\frac{k_s B_0^* Z'}{A_0^*}$$

$$+\frac{4}{3}\frac{k_p \frac{\partial P_0}{\partial Z}}{P_0^*} Z')] = 0$$

This system has the following form:

$$A\begin{pmatrix}\frac{\partial Q'}{\partial x^*}\\ \frac{\partial Z'}{\partial x^*}\end{pmatrix} = B\begin{pmatrix}Q'\\ Z'\end{pmatrix}$$

$$\begin{pmatrix}\frac{\partial Q'}{\partial x^*}\\ \frac{\partial Z'}{\partial x^*}\end{pmatrix} = A^{-1} B\begin{pmatrix}Q'\\ Z'\end{pmatrix}$$

Put $X = \begin{pmatrix}x_{11} & x_{12}\\ x_{21} & x_{22}\end{pmatrix} = A^{-1}B$

After integration the equation becomes:

$$\begin{pmatrix}Q'\\ Z'\end{pmatrix}_{x^*} = e^{\int X\,dx^*}\begin{pmatrix}Q'\\ Z'\end{pmatrix}_0$$

If X is supposed constant with x^* then:

$$\begin{pmatrix}Q'\\ Z'\end{pmatrix}_{x^*} = e^{X x^*}\begin{pmatrix}Q'\\ Z'\end{pmatrix}_0$$

Lets look at eigenvalues of X:

$$\lambda_{1,2} = \frac{x_{22} \pm \sqrt{x_{22}^2 - 4k_s L_0^* s\, x_{21}}}{2}$$

Put:

$$D' = \begin{pmatrix} e^{\lambda_1 x^*} & 0\\ 0 & e^{\lambda_2 x^*}\end{pmatrix}$$

And $P = \begin{pmatrix} -k_s L_0^* s & -k_s L_0^* s\\ \lambda_1 & \lambda_2\end{pmatrix}$

So

$$\begin{pmatrix}Q'\\ Z'\end{pmatrix}_{x^*} = P\, D'\, P^{-1}\begin{pmatrix}Q'\\ Z'\end{pmatrix}_0 = A\begin{pmatrix}Q'\\ Z'\end{pmatrix}_0$$

with:

$$a_{11} = \frac{\lambda_1 e^{\lambda_2 x^*} - \lambda_2 e^{\lambda_1 x^*}}{\lambda_1 - \lambda_2}, \quad a_{12} = \frac{kL_0^* s\,(e^{\lambda_2 x^*} - e^{\lambda_1 x^*})}{\lambda_1 - \lambda_2}$$

$$a_{21} = \frac{\lambda_1 \lambda_2 (e^{\lambda_1 x^*} - e^{\lambda_2 x^*})}{kL_0^* s\,(\lambda_1 - \lambda_2)}, \quad a_{22} = \frac{\lambda_1 e^{\lambda_1 x^*} - \lambda_2 e^{\lambda_2 x^*}}{\lambda_1 - \lambda_2}$$

The boundary conditions for a subcritical flow are the upstream discharge and the downstream level. These two variables are considered as inputs. The downstream discharge and the upstream level are the outputs.
Let's define the transfer matrix M of the reach as:

$$\begin{pmatrix}Q'_{x^*}\\ Z'_0\end{pmatrix} = M\begin{pmatrix}Q'_0\\ Z'_{x^*}\end{pmatrix}$$

with:

$$m_{11} = \frac{(\lambda_1-\lambda_2)\, e^{(\lambda_1+\lambda_2)x^*}}{\lambda_1 e^{\lambda_1 x^*} - \lambda_2 e^{\lambda_2 x^*}}, \quad m_{12} = \frac{kL_0^* s(e^{\lambda_2 x^*} - e^{\lambda_1 x^*})}{\lambda_1 e^{\lambda_1 x^*} - \lambda_2 e^{\lambda_2 x^*}}$$

$$m_{21} = \frac{-\lambda_1\lambda_2 (e^{\lambda_1 x^*} - e^{\lambda_2 x^*})}{kL_0^* s\,(\lambda_1 e^{\lambda_1 x^*} - \lambda_2 e^{\lambda_2 x^*})}, \quad m_{22} = \frac{\lambda_1 - \lambda_2}{\lambda_1 e^{\lambda_1 x^*} - \lambda_2 e^{\lambda_2 x^*}}$$

This modelisation has the advantage of keeping the distributed parameter system characteristics and therefore the infinite state space dimension.

4. STUDY OF THE TRANSFER MATRIX

Frequency reach responses :

The study focuses on the two transfer functions m_{11} and m_{22}. The m_{11} transfer function characterises the influence of a upstream water release to the downstream discharge. The m_{22} transfer function modelizes the effect of a downstream variation of level (due to a gate movement) on the upstream level.
In the case of Type 1 (Figure 1) and Type 5 reaches (Figure 2), Bode response was simulated for the transfer function between upstream and downstream discharges.

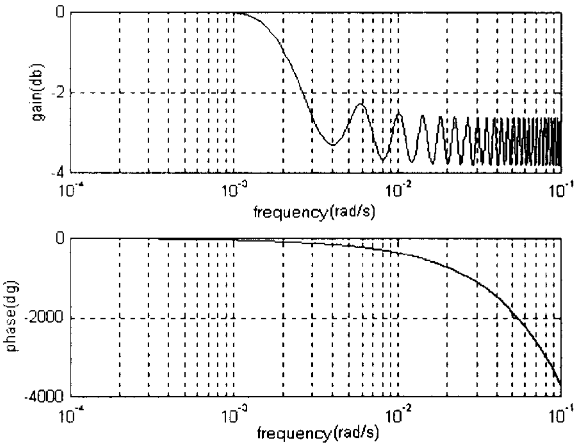

Figure 1 : Type 1, Bode response for m_{11}

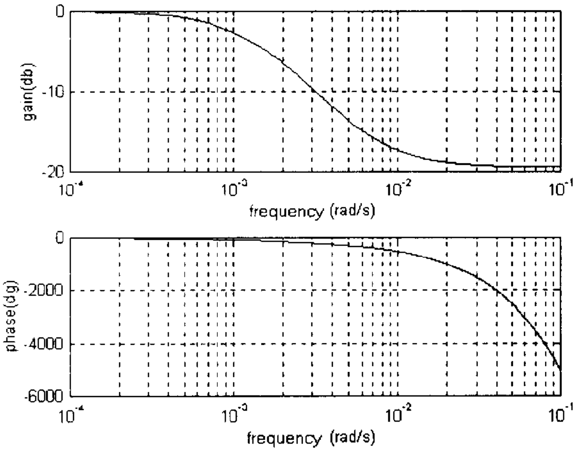

Figure 2 : Type 5, Bode response for m_{11}

For Type 1 some amplitude peaks are obtained for high frequencies as observed by Schuurmans [6]. These peaks are not observed for Type 5 . These results were expected due to the value of η for the two types.

For Type 1 Bode response was simulated after neglecting inertial terms. Peaks have disappeared and the response is very near to the one obtained by a first or a second order transfer function.

Figure 3 for Type 1 and Figure 4 for Type 5 show the Bode response for the transfer function between downstream and upstream level.

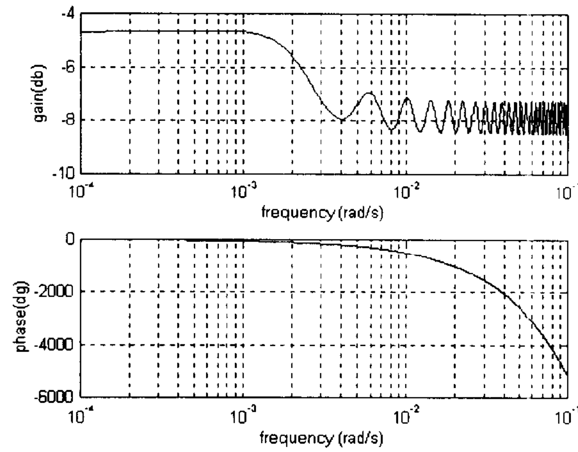

Figure 3 : Type 1, Bode response for m_{22}

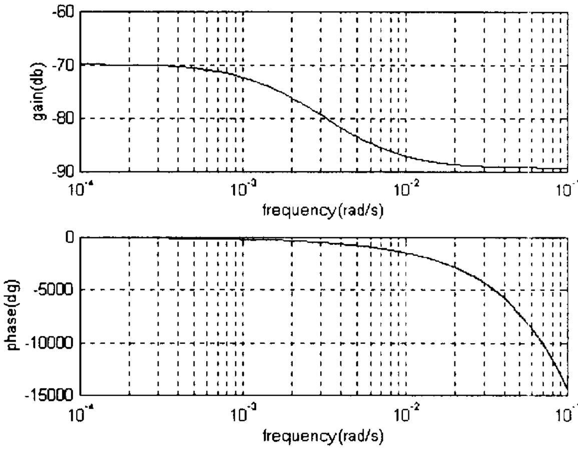

Figure 4 : Type 5, Bode response for m_{22}

For Type 1 some amplitude peaks are obtained for high frequencies as observed for the discharge. For Type 5 downstream level perturbations have quite no influence on upstream level, the static magnitude is very low.

Time responses :

This evaluation is done using a model based on a finite difference approximation of Saint Venant equations by Preissmann's scheme [7] . This model is modified to be able to integrate the linear set of equations.

Simulations are done for a step of upstream discharge of 5% of Q_M for the two types. The dimensionless computation step in space is 0.1 and the time step is taken to have the same Courant number for each type.

Simulations done for Q_M show that there is no visible difference between linear and complete non-linear model for the two types of reaches. Simulations done for $0.5*Q_M$ show that the linear model is quite good but with a delay shorter than for the non-linear response. The results are shown for Type 1 (Figure 5) and for Type 5 (Figure 6).

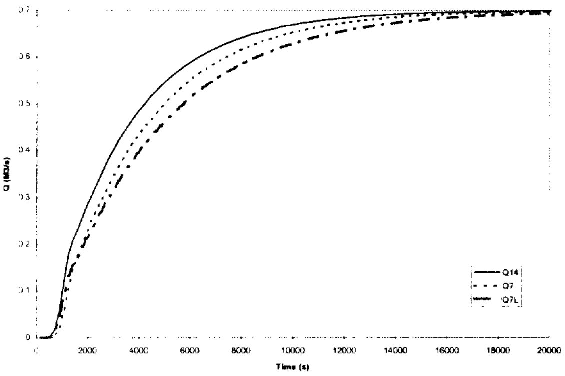

Figure 5 : Type 1, Upstream discharge step response

Where Q14 and Q7 are the non-linear response for a discharge of 14 and 7 m^3/s and Q7L is the linear response for a discharge of 7 m^3/s.

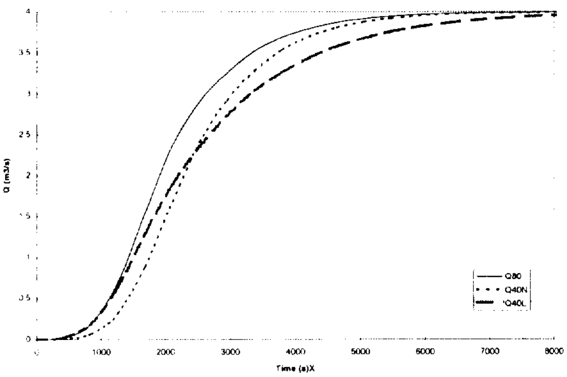

Figure 6 : Type 5, Upstream discharge step response

Where Q80 and Q40 are the non-linear response for a discharge of 80 and 40 m^3/s and Q40L is the linear response for a discharge of 40 m^3/s.

The same simulations are done for a canal with five pools and gated cross-structures with submerged flow conditions. The dimensionless target level upstream each gate is 1 and the discharge is 0.5*Q_M. Simulations are conducted with three kind of models : the full Saint Venant non-linear model (N), linear model for reaches and non-linear model for the cross structures (LN), linear model for the reaches and the cross structures (L).
The results are shown (Figure 7) for Type 1 and (Figure 8) for Type 5.

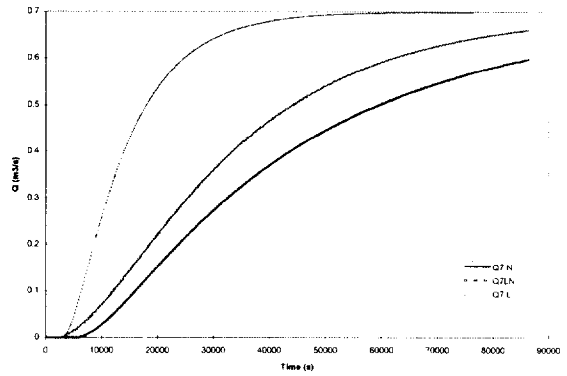

Figure 7 : Type 1, Upstream discharge step response for 5 reaches

Where Q7N, Q7LN, Q7L are downstream canal responses for a discharge of 7 m^3/s for the three kind of models.

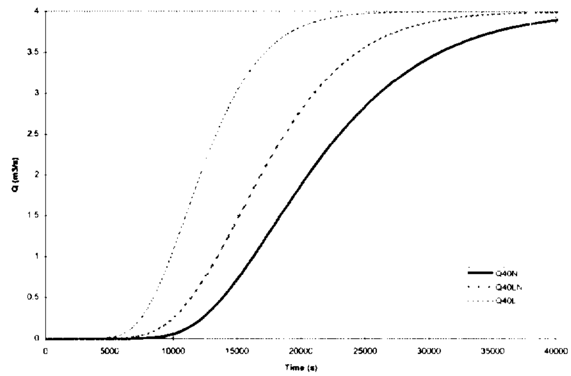

Figure 8 : Type 5, Upstream discharge step response for 5 reaches

Where Q40N, Q40LN, Q40L are downstream canal responses for a discharge of 40 m^3/s for the three kind of models.

The simulations show that discrepancies between (N) and (L) or (NL) is bigger for Type 5 than for Type 1. This is due to the backwater curve that is closer to the uniform level for a short reach like Type 1 than for a long reach like Type 5.
The (LN) model is very close to the (N) model for Type1 and is also better than the (L) model for Type5. The discrepancies between full Saint Venant model and (L), are mainly due to the linearization of cross-structure laws.
The results show that the transfer matrix is very accurate near the steady flow used for the linearisation but also that it is still valid far from these hydraulic conditions.

5. CONCLUSION

The knowledge of the reach transfer matrix opens new possibilities for the design of canal system controller.

The accuracy of the transfer matrix was investigated using a full Saint Venant model. The results show that the reach transfer matrix is accurate even if the water profile is far from the steady state used for linearization. The main discrepancies are due to cross-structure linearisation. To design a canal system controller, the engineer has to pay special attention on this particular point.

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