WAVE MOTION STABILITY FOR COUPLED CANAL POOL-*AMIL* GATE SYSTEMS

by : **Ramirez-Luna[1] J., Baume[2] J. P., De León-Mojarro[3] B., Ruiz-Carmona[3] V. M., Sau[4] J.**

[1] Researcher Ingeneer, DIVISION OF IRRIGATION & DRAINAGE, IMTA, 8534, Jiutepec, Morelos, Mexico.
[2] Researcher Ingeneer, IRRIGATION DIVISION, CEMAGREF-MTP, 361, rue J-F Breton BP 5095, 34033 Montpellier, France.
[3] Professor, DIVISION OF IRRIGATION & DRAINAGE, IMTA, 8534, Jiutepec, Morelos, Mexico.
[4] Professor. UNIVERSITE Claude Bernard Lyon I, ISTIL Mecanique-Automatique, Bât 201, Campus Doua, 43 Blvd 11 Nov. 1918, 69622 Villeurbanne. France.
email : hperea@riego.imta.mx, Tel/Fax : +52 73.19.40.41

ABSTRACT

The studied system is an irrigation canal with hidromechanic *AMIL* gates. This gate is used in order to control (proportional controler) a local upstream water level [1] [2]. These gates are associated with an unpstream water distribution method. The stability study was conducted as follows: first) on a coupled canal pool - *AMIL* gate, such a system was named subsystem, and second) on two coupled subsystems. A type turnout perturbation is supossed to be in the upstream side close to the gate.

The dimensionless *S' Vcnant* [3] equations are linearized around a normal steady flow. Two dimensionless numbers are important: the *Froude* number and the dimensionless length. By *Laplace* transform, a feasible system is established. By integration, the distributed parameter system was obtained. It represents the canal pool dynamics [4]. The dynamics of the *AMIL* gate is supossed as succesion of steady states [5]. The interested transcendent transfer function is established by coupling the pool dynamics representation and the linearized discharge equation of *AMIL* gate.

The stability study consists in determining the analicity of the different transcendent input-output transfer in the *Right Half Plan* (*RHP*) [6] [7]. The resulting stability condition depends on the dimensionless numbers and the linearization parameters of the discharge equation.

- In the first case, it was established a sufficient and necessary condition, so that, the coupled canal pool-*AMIL* gate is inconditionally stable.
- In the second case, it was established a sufficient condition. Thus this condition has been evaluated by using different geometric and hydraulic characteristics for trapecial test canal [4]. This was retained: 1) the instability problems become from the interaction of the hydraulic information contained in both subsystems and that 2) this interaction is important when the wave celerity is high.

As a result, we propose an easy-use stability sufficient condition for coupled subsystems. A general method was used in order to study the wave motion stability for simple and coupled subsystems. This method can be directly applied when studying the stability problems of all canal pool - motorized hidraulic structure (controler) system. Obtained conditions can be used like design norms of irrigation canals.

Keywords : regulation methods, hydraulics, mechanic equations, signal, automatica.

1. INTRODUCTION

In Irrigation Districts where water distribution is by gravity two aspects are present: *i)* the transfer of a water volume in a given instant and ii) the precision of water deliveries (takeoffs) in turnouts. First, the dynamics of this transfer is characterized by a long time delay, which is a function of the distance between water sources and turnouts. A canal pool is the piece of canal separated by two *AMILs*. Canal pools react as buffer reservoirs to reply quickly discharge and water perturbations in turnouts. In the other hand, the precision of the water distribution in turnouts depends on maintaining a constant water level in each turnout. To illustrate this problem, we consider a principal canal used to transfer a discharge with an upstream local control. Turnouts are located at the upstream of the gate in an upstream local control [8]. See Figure 1.

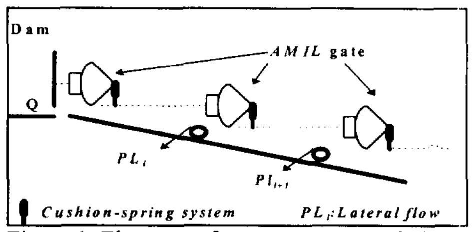

Figure 1. Elements of « upstream » regulation

AMILs maintain the upstream water level quasi-constant (a float is placed at the upstream of the gate) and these gates are widely used in irrigation canal nets because they improve strongly the water distribution. Furthermore, they are robust and well-adapted to countries where energy economy is a priority and to places where power energy is not available.

This study has been done in collaboration with *CEMAGREF* (Public Water Research Institute: Irrigation Division) at Montpellier, France. The principal objectives are: *i)* to study the stability of coupled canal pool - *AMIL* systems and *ii)* to deduce norms of canal design in presence of *AMILs*. This model can also be inserted in a hydraulics model package for surface flow. The *AMIL* dynamics is first established and after that one of the canal pool.

2. DYNAMIC OF *AMIL* GATES

Geometrically, *AMILs* are radial gates of trapezoidal section. An upstream toroid float is joined to cylindrical gate-leaf. Elements, such as dash-pot, introduce friction into the system and are used to improve the gate stability. For *AMILs*, a spring-cushion mechanism is fixed between a gate arm and a stand-by bar (see Figure 2). Forces acting on the system (used for computing the torques with respect to the gate axis) are: i) the proper gate weight and the counterweights and ii) hydrostatic force acting on the surface of toroid float. The gate-leaf surface is cylindrical, the result of hydrostatic forces passes throughout axis and its torque vanishes [9].

Notation : α opening angle of the gate, α_0 opening angle associated to a water controlled level at the axis height $\mathbf{z}_1=0$ defined by the relation $\mathbf{z}_{min}/z_{max}=-\sin(\alpha_o)/\sin((\alpha_{max}-\alpha_o)$, z_{max} water level with respect to the axis (associated with a maximal

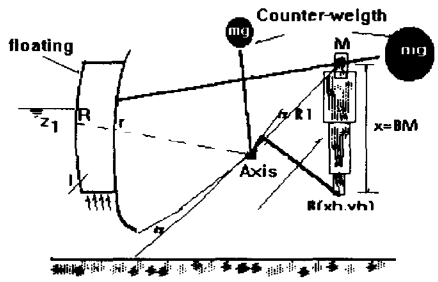

Figure 2 ***AMIL*** **gate**

opening), z_{min} water level with respect to the axis (associated with a closed gate), ρ water density, α_{max} maximal opening angle, g gravity, $\mathbf{z}_1$ upstream water level, $D=z_{max}-z_{min}$, x shock absorber guide, xb and yb point of coordinates where the shock absorber is fixed, $R1$ radius to M point of shock absorber, I inertial moment, t time, l float wide, R external float radius, r internal float radius.

Dynamic behavior depends on the «adjustment by counterweights». Adjustments are carried out in making extreme equilibriums where the counterweights weight is adequately distributed into adapted cavities. Extreme equilibriums determine a maximal interval (decrement D) in which the upstream water controlled level must vary, each opening is determined by this water level variation. Extreme equilibriums correspond to *1)* a maximal opening of the gate when a superior water level z_{max} equals an upper water controlled level and *2)* a closed gate has to coincide when an inferior water level z_{max}-D equals a lower water controlled level z_{min}.

The hydrostatic force on the float and weight forces produce the motor torque. Determination of the torque value that corresponds to extreme equilibriums, makes possible to establish the relation between opening angle and the water controlled level.

This torque is

$$C_M = K\left(z_1 - \frac{D}{\sin(\alpha_{max} - \alpha_0) + \sin(\alpha_0)}\sin(\alpha - \alpha_0)\right) \quad 1)$$

$$K = \frac{1}{2}\rho g l\left(R^2 - r^2\right)$$

where K results from the computation of the force (Archimede) acting on the float considering that the hydrostatic pressure forces (result) acting only on the lowest float surface. In every static equilibrium ($C_M=0$), the relation *1)* establishes the opening of the gate as a function of a water level $\mathbf{z}_1$, such as $z_{min} \leq z_1 \leq z_{max}$ represents the water controlled level in a stationary regime.

Applying the moment theorem, the dynamic equation is:

$$I\frac{d^2\alpha}{dt^2} + F\frac{d\alpha}{dt} - K\left(\frac{D}{\sin(\alpha_{max} - \alpha_0) + \sin(\alpha_0)}\right)\sin(\alpha - \alpha_0) = Kz_1 \quad 2)$$

where I is the inertial moment, F is a spring-cushion coefficient. We have a second order system in α and $\mathbf{z}_1$.

As presented in [5], we can say that *AMILs* follow a slow variations of the water controlled level according to a static relation. Several responses have confirmed the role of « low-pass » filter of the cushion-spring mechanism. Element of energy dissipation as well as the element ensuring the gates stability is also the cushion-spring mechanism. The time constants of the gate dynamics are greater than these ones of canal pool dynamics.

3. DISCHARGE EQUATION

The *AMIL* dynamic behavior could be represented as a sequence of steady states. *Gec-Alsthom Inc.* proposes an experimental discharge equation type : $Q(\alpha) = C(\alpha)Q_{max}$, where $Q_{max} = CdS(w_{max})\sqrt{2g(Z_L - Z_0)}$ is the classical discharge equation for a maximal openning and $C(\alpha)$, the partial openning coefficient, is $C(\alpha)=C_1\sin(\alpha)-C_2\cos(\alpha)+C_3$, $C_1=\left(0.2295Dv/R_v\right)^{-1}\sqrt{1-(0.45Dv/R_v)^2}$, $c_2=1.9608$ and $c_3=c_2$. From above, openning variable w is a function of α and Z_L is the upstream gate water level (downstream canal pool water level) and Z_o is downstream gate water level (the upstream consecutive canal pool water level). D_v and R_v are respectively gate-leaf mean width evaluated when z_l equals the gate axis high and the gate radius. *AMIL* works on free and submerged flow.

For a first order development around a steady point (X_{0s}):

$$\delta Q(t) = \left.\frac{\partial Q}{\partial \alpha}\right|_{X_{0s}}\delta\alpha(t) + \left.\frac{\partial Q}{\partial Z_L}\right|_{X_{0s}}\delta Z_L(t) - \left.\frac{\partial Q}{\partial Z_0}\right|_{X_{0s}}\delta Z_{0i}(t)$$

$$,Q(s) = C_4\alpha(s) + C_5 Z_L(s) - C_6 Z_0(s) \quad 3)$$

Outcome (3) comes from linearization, *Laplace* transform aplication and the coefficients C_4, C_5, C_6 evaluation. In equation (2), $z_l(s)$ corresponds to $Z_L(s)$. From the static model (1) and for a particular case, $\alpha(s)$ can be expressed in termes of $z_l(s)$. So, in terms of $Z_L(s)$:

$$Q(s) = c_{11}Z_L(s) - c_{12}Z_0(s),$$

$$c_{11} = \left(\frac{C_4}{\cos(\alpha|_{X_{0s}})} + C_5\right),\ c_{12} = C_6 \quad 4)$$

This formulation is well-adapted in the case where the gate time constants are less than these ones of the canal pool dynamics. Notice that different constants must be determined from dimensionless equations. It is clear that linearization constants presented in stability criteria evaluation become from the dimensionless equations.

4. DYNAMIC OF THE CANAL POOL

According to [4], the transfer matrix is

$$G(s) = \frac{1}{\lambda_1(s)e^{\lambda_1(s)x^*} - \lambda_2(s)e^{\lambda_2(s)x^*}}\begin{bmatrix} (\lambda_1(s)-\lambda_2(s))e^{(\lambda_1(s)+\lambda_2(s))x^*} & ksL_0^* s(e^{\lambda_2(s)x^*} - e^{\lambda_1(s)x^*}) \\ \frac{(\lambda_1(s)\lambda_2(s))}{ksL_0^* s}(e^{\lambda_2(s)x^*} - e^{\lambda_1(s)x^*}) & (\lambda_1(s)-\lambda_2(s)) \end{bmatrix} \quad 5)$$

where x^* is the canal pool dimensionless length and $(\lambda_1(s)$ and $\lambda_2(s))$ are the proper values of $A(s)$.

$$\lambda_{1,2} = \frac{1}{2}\frac{ksL_0^*\beta_2 s - \beta_5}{\beta_3} \pm \frac{1}{2}\sqrt{\Delta(s)},$$

$$A(s) = \begin{bmatrix} 0 & -ksL_0^* s \\ -\frac{s+\beta}{\beta_3} & \frac{ksL_0^*\beta_2 s - \beta_5}{\beta_3} \end{bmatrix},\quad F_r = \frac{Q_r}{S_r}\frac{1}{\sqrt{gy_r}},\quad \chi = \frac{x_r J_r}{y_r}$$

These latter equations are respectively the reference *Froude* number and the dimensionless reference length. Others parameters are:

$$\beta_2 = 2ksL_0^*\frac{Q_0^*}{S_0^*},\quad \beta_3 = \left[\frac{S_0^*}{F_r^2} - ksL_0^*\frac{Q_0^{*2}}{S_0^{*2}}\right]$$

$$\beta = 2\left[\frac{Q_0^*}{S_0^{*2}}\frac{\partial S_0^*}{\partial x} - \frac{\chi}{F_r^2}\frac{S_0^*}{Q_0^*}J_0^*\right] \quad 6)$$

$$\beta_5 = -2\,ksL_0^{\bullet}\,\frac{Q_0^{\bullet 2}}{S_0^{\bullet 3}}\frac{\partial S_0^{\bullet}}{\partial x} - \frac{ksL_0^{\bullet}}{F_r^2}\frac{\partial Z_0^{\bullet}}{\partial x} +$$

$$\frac{7}{3}\frac{\chi}{F_r^2}ksL_0^{\bullet}J_0^{\bullet} - \frac{4}{3}\frac{\chi}{F_r^2}kpP_rR_0^{\bullet}J_0^{\bullet}$$

and $\sqrt{\Delta(s)} = \sqrt{\left(\frac{ksL_0^{\bullet}\beta_2 s - \beta_5}{\beta_3}\right)^2 - 4\left(\frac{ksL_0^{\bullet} s(\beta_1 s - \beta)}{\beta_3}\right)}$

In the above equations, subscript 0 is related to steady values of hydraulic variables. For a trapecial canal section, reference variables are: canal width $l_r=L_b+2my_r$, section $S_r=y_r(L_b+my_r)$, wetted perimeter $P_{mr}=L_b+2(1+m^2)^{1/2}{}_{yr}$, and hydraulic radius $R_r=S_r/P_{mr}$, with L_b bottom canal width and m the bank slope. Others variables are: $ks=l_ry_r/S_r$, $kp=y_r/P_{mr}$, $dP_{mo}/dz=2(1.0+m^2)^{1/2}$, where, shape factor ks, kp perimeter factor and derivative of wetted perimeter with respect to the water level, respectively. The reference vertical scale is $y_r=y_n$, the normal water level and the reference horizontal scale is x_r, the canal pool length. Reference discharge Q_r corresponds to normal discharge. Equation (5) is the dynamics of the canal pool and it represents the distributed parameter system.

5. INTERPRETATION OF THE MATRIX *G(S)*

To interpretate physically the transfer matrix, the following hypothesis are made: *i)* canal is rectangular ($ks=1$), *ii)* friction rate J and *iii)* canal bed slope I are negligible. Such hypothesis allow us to a particular formulation of the proper values. Let's F_r be $F_r=v_r/c_r$.

The proper values are: $\lambda_1(s) = \frac{F_r}{1-F_r}s = -\frac{v_r}{v_r-c_r}s$,

$$\lambda_2(s) = -\frac{F_r}{1+F_r}s = -\frac{v_r}{v_r+c_r}s.$$

Dimensionless velocities for positive and negative characteristics waves [10] [11] are identified. For a subcritical regime, $\lambda_{1(s)}$ is positive and $\lambda_{2(s)}$ is negative. The product $\lambda_1(s)x^*$ is the dimensionless characteristic time for an up to downstream wave and $\lambda_2(s)x^*$ respectively for a down to upstream wave. Exponential terms $e^{(\lambda 1(s))}$ and $e^{(\lambda 2(s))}$ represents, in *Laplace* space, the delay terms.

The proper value ratio gives: $\frac{\lambda_2(s)}{\lambda_1(s)} = -\frac{1-F_r}{1+F_r} = \frac{v_r-c_r}{v_r+c_r}$, is the cocient of dimensionless characteristic times for a up to down and down to upstream waves. This ratio is less than one and negative under regime subcritical condition. Addition of proper values is: $\lambda_1(s)+\lambda_2(s) = \frac{F_r}{1-F_r}s - \frac{F_r}{1+F_r}s = -v_r\left(\frac{1}{v_r-c_r}+\frac{1}{v_r+c_r}\right)s$, this is the dimensionless characteristic time of a full period wave.

In the general case, non negligeable friction rate and canal bed slope are different from zero and the canal section is not rectangular, the proper values keep the same meaning but including the terms of wave damping and deformation. The above analysis allow us to write the transfer matrix in the following form which simplifies the analysis of the coupled systems stability.

$$G(s) = \left(\lambda_1(s)\left(1-\frac{\lambda_2(s)}{\lambda_1(s)}e^{(\lambda_1(s)-\lambda_2(s))x^*}\right)\right)^{-1}\begin{bmatrix} (\lambda_1(s)-\lambda_2(s))e^{\lambda_2(s)x^*} & -ksL_0^{\bullet}s(1-e^{(\lambda_1(s)-\lambda_2(s))x^*}) \\ -\frac{\lambda_1(s)\lambda_2(s)}{ksL_0^{\bullet}s}(1-e^{(\lambda_1(s)-\lambda_2(s))x^*}) & (\lambda_1(s)-\lambda_2(s))e^{\lambda_1(s)x^*} \end{bmatrix} \quad 5')$$

6. STABILITY OF COUPLED CANAL POOL - *AMIL* GATE SYSTEMS

Because of the distributed parameter character of the system, the stability criteria are difficult to be obtained. To obtain these criteria, the basic automatica theorem is applied, so that, the transfer functions must be analytical and bounded in the *RHP*. The transfer function are obtained from the canal pool dynamics contained in *G(s)* (5) and that one of the gate in *Q(s)* (4). A single coupled canal pool -*AMIL* is named subsystem. The transfer functions of subsytems and coupled subsytems are obtained from the hydraulic variable continuity.

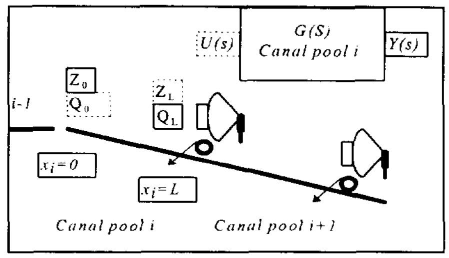

Figure 3 Canal pools and *AMIL* gates. Inputs *U(s)* and outputs *Y(s)*

For two coupled subsystems, notation is as follows: 1) ***O*** is canal pool initial abscissa , ***L*** canal pool final abcissa. ***O*** and ***L*** are used to indicate position of some variables (for example P_{Li}, is lateral discharge at downstream position of *i-th* canal pool. An *AMIL* controls a local upstream water level.

The general representation of a subsystem is:

$$Z_{Li}(s) = T(s)*U(s) + W(s)*P_L(s) \qquad 7)$$

where $U(s) = [Qo_i, Zo_{i+1}]^T$ $\quad P_L(s) = [PL_i]$. *U(s)* is input vector (upstream discharge and downstream water level in the canal pool), $P_L(s)$ is the lateral flow or perturbation vector. $Z_{Li}(s)$ is the output (upstream local water level).

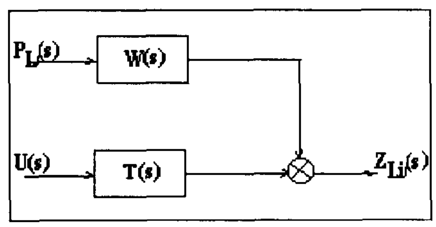

Figure 4 Scheme of linearized system

Figure 4 presents this linearized subsystem. The transfer function of the controlled upstream water level Z_{Li} is: $Z_{Li} = T_1(s)\,Qo_i + T_2(s)\,Zo_{i+1} + W_1(s)\,P_{Li}$, where:

$$T_1(s) = \frac{\sqrt{\Delta(s)}e^{\lambda_2(s)}}{c_{11}f_1(s)F_1(s)}, \qquad T_2(s) = \frac{c_{12}\lambda_1(s)F_2(s)}{c_{11}f_1(s)F_1(s)},$$

$$W_1(s) = -\frac{\lambda_1(s)F_2(s)}{c_{11}f_1(s)F_1(s)}; \qquad F_1(s) = 1-\frac{f_2(s)}{f_1(s)}e^{-\sqrt{\Delta(s)}},$$

$$F_2(s) = \left(1-\frac{\lambda_2(s)}{\lambda_1(s)}e^{-\sqrt{\Delta(s)}}\right); \alpha_0 = 2\frac{S_0^{\bullet}}{Q_0^{\bullet}} \qquad \text{with:}$$

$$\Delta(s) = (a_1-b_1)^2 + 2(a_1+b_1)(a_2+b_2)s + (a_2+b_2)^2s^2,$$

$$r_1 = 2\,ksL_0^{\bullet}\frac{Q_0^{\bullet}}{S_0^{\bullet}}\left(\frac{S_0^{\bullet}}{F_r^2} - ksL_0^{\bullet}\left(\frac{Q_0^{\bullet}}{S_0^{\bullet}}\right)^2\right)^{-1},$$

$$a_1 = \frac{\alpha_0}{2}\left(1 + \alpha_0/r_1\right)^{-1/2}\left(r_1\alpha_2\left(1 + \sqrt{1 + \alpha_0/r_1}\right) + \frac{1}{2}\alpha_0\alpha_4\right),$$

$$\alpha_4 = J_0^* \frac{\chi}{F_r^2} \quad \alpha_2 = \frac{1}{4}\gamma_2$$

$$b_1 = \frac{\alpha_0}{2}\left(1 + \alpha_0/r_1\right)^{-1/2}\left(r_1\alpha_2\left(1 - \sqrt{1 + \alpha_0/r_1}\right) + \frac{1}{2}\alpha_0\alpha_4\right),$$

$$a_3 = \frac{ksL_0^*}{c_{11}}, \quad a_2 = \frac{1}{2}r_1\left(1 + \sqrt{1 + \alpha_0/r_1}\right)$$

$$b_2 = \frac{1}{2}r_1\left(\sqrt{1 + \alpha_0/r_1} - 1\right),$$

$$\gamma_2 = \left(\chi + \frac{\chi}{3}J_0^*\left(7 - 4\frac{S_0^*}{ksL_0^*}\frac{Kp}{P_{m0}^*}\frac{\partial(P_{mo})}{\partial z}\right)\right)F_r^{-2},$$

$$\lambda_1(s) = \frac{1}{2}\left((a_1 - b_1) + (a_2 - b_2)s + \sqrt{\Delta(s)}\right),$$

$$\lambda_2(s) = \frac{1}{2}\left((a_1 - b_1) + (a_2 - b_2)s - \sqrt{\Delta(s)}\right),$$

$$f_1(s) = \frac{1}{2}\left((a_1 - b_1) + (a_2 - b_2 + 2a_3)s + \sqrt{\Delta(s)}\right)$$

$$f_2(s) = \frac{1}{2}\left((a_1 - b_1) + (a_2 - b_2 + 2a_3)s - \sqrt{\Delta(s)}\right).$$

Subsystem stability

- The subsystem is stable if the transfer functions have not unstable poles.

Our objective is to show in *RHP*, that denominators of T_1, T_2 and W_1 do not vanish. So, $f_1(s)$ and $F_1(s)$ do not vanish on *RHP*.

From $F_1(s) = 1 - \frac{f_2(s)}{f_1(s)}e^{-\sqrt{\Delta(s)}}$, by majoring we show $\left|\frac{f_2(s)}{f_1(s)}\right|\left|e^{-\sqrt{\Delta(s)}}\right| < 1$ and from $f_1(s)$, thus $\mathrm{Re}(f_1(s))$ is always positive, its module does not vanish.

As conclusion these functions are analytical. It has been demonstrated that a subsystem is unconditionally stable. Furthermore, possible problems of unstability presents in the coupled canal pool flow (5') - *AMIL* (4) dynamics are characterized by discharge and surface wave propagation. So that, is the wave energy which produces the interaction (weak or strong) between *AMILs*. In this sense it is interesting to study the stability of coupled subsystems.

Two coupled subsystem stability

The above results are physically logic. In presence of a finite energy perturbation, the subsystem dynamics is stable in the sense that the flow is a non negligible friction flow (energy dissipation, in spite of gravitational energy contribution) and the gate is an element of energy dissipation.

The transfer function representation is as follows: In (8), $U(s)$ is the input vector, $P_L(s)$ is the perturbation vector (lateral flows). $Z_{Li}(s)$ is the interested output vector, so $U(s) = [Qo_i \; Zo_{i+2}]^T \quad P_L(s) = [P_{Li} \; P_{Li+1}]^T$.

The input-output transfers can be written as:

$$Z_{Li}(s) = T_{11}(s)Qo_i + T_{12}(s)Zo_{i+2} + P_{11}(s)P_{Li} + P_{12}(s)P_{Li+1} \qquad 8)$$

$$Z_{Li+1}(s) = T_{21}(s)Qo_i + T_{22}(s)Zo_{i+2} + P_{21}(s)P_{Li} + P_{22}(s)P_{Li+1}$$

where:

$$T_{11}(s) = \frac{\sqrt{\Delta(s)}e^{\lambda_2(s)}\left((c_{11}f_1(s)F_1(s))(\lambda_1(s)F_2(s))\right)}{(c_{11}f_1(s)F_1(s))^2\lambda_1(s)F_2(s)D_1(s)} + \frac{\sqrt{\Delta(s)}e^{\lambda_2(s)}\left(c_{12}\Lambda_{21}(s)F_3(s)) + c_{12}\Delta(s)e^{-\sqrt{\Delta(s)}}\right)}{(c_{11}f_1(s)F_1(s))^2\lambda_1(s)F_2(s)D_1(s)}$$

$$T_{12}(s) = \frac{c_{12}^2\sqrt{\Delta(s)}e^{-\lambda_1(s)}\lambda_1(s)F_2(s)}{(c_{11}f_1(s)F_1(s))^2D_1(s)},$$

$$P_{11}(s) = -\frac{(c_{11}f_1(s)F_1(s))(\lambda_1(s)F_2(s)}{(c_{11}f_1(s)F_1(s))^2D_1(s)} - \frac{c_{12}\Lambda_{21}(s)F_3(s)) + c_{12}\Delta(s)e^{\lambda_2(s)}}{(c_{11}f_1(s)F_1(s))^2D_1(s)}$$

$$P_{12}(s) = -\frac{c_{12}\sqrt{\Delta(s)}e^{-\lambda_1(s)}\lambda_1(s)F_2(s)}{(c_{11}f_1(s)F_1(s))^2D_1(s)},$$

$$T_{21}(s) = \frac{c_{11}\Delta(s)e^{2\lambda_2(s)}}{(c_{11}f_1(s)F_1(s))^2D_1(s)}$$

$$T_{22}(s) = c_{12}\frac{(c_{12}\Lambda_{21}(s)F_3(s)ksL_0^*\,sF_3(s) + \lambda_1(s)F_2(s)c_{11}f_1(s)F_1(s))}{(c_{11}f_1(s)F_1(s))^2D_1(s)}$$

$$P_{21}(s) = -\frac{c_{11}\lambda_1(s)F_2(s)\sqrt{\Delta(s)}e^{\lambda_2(s)}}{(c_{11}f_1(s)F_1(s))^2D_1(s)},$$

$$P_{22}(s) = -\frac{(c_{12}\Lambda_{21}(s)F_3(s)ksL_0^*\,sF_3(s) + \lambda_1(s)F_2(s)c_{11}f_1(s)F_1(s))}{(c_{11}f_1(s)F_1(s))^2D_1(s)},$$

$$F_1(s) = 1 - \frac{f_2(s)}{f_1(s)}e^{-\sqrt{\Delta(s)}} \quad F_2(s) = 1 - \frac{\lambda_2(s)}{\lambda_1(s)}e^{-\sqrt{\Delta(s)}},$$

$$F_3(s) = 1 - e^{-\sqrt{\Delta(s)}} \quad \Lambda_{21}(s) = \frac{s + \beta}{\beta_3}$$

and

$$D_1(s) = 1 + \frac{c_{12}}{c_{11}}ksL_0^*\frac{s}{\lambda_1(s)}\frac{(F_3(s))^2}{F_1(s)F_2(s)} * \left\{\frac{\Lambda_{21}(s)}{f_1(s)} + \frac{\Delta(s)}{c_{11}(f_1(s))^2}\frac{e^{-\sqrt{\Delta(s)}}}{F_1(s)F_3(s)}\right\}$$

In the above section, it has been presented some function properties ($F_1(s)$, $f_1(s)$ and $\lambda_{1,2}(s)$) on *RHP* ($Re(s)\geq 0$). In particular, such functions are analytical on *RHP*. As before, it has to be established the conditions for what transfer functions $T_{ij}(s)$ and $P_{ij}(s)$ ($i, j=1,2$) have not unstable poles. Functions $f_1(s)$, $\lambda_1(s)$, $F_1(s)$, $F_2(s)$ and $D_1(s)$ intervene in transfer function denominators (8). The functions $f_1(s)$, $\lambda_1(s)$, $F_1(s)$, $F_2(s)$ are analytical on *RHP*. So that, eventual unstability of coupled subsystems comes from the properties of the function $D_1(s)$.

After this, a sufficient condition is given in order to $1/D_1(s)$ is analytical on *RHP*. Let's a_3, a_4 and a_5 be $a_3 = \frac{ksL_0^*}{c_{11}}$, $a_4 = c_{12}a_3$ and $a_5 = \frac{1}{c_{11}}$. Let's $a_1 > b_1$ et $a_2 > b_2$, a_4 and a_5, positive reals such that $1/D_1(s)$ is analytical if:

$$|D1(s)| = \left| a_4\frac{s}{\lambda_1(s)}\frac{(F_3(s))^2}{F_1(s)F_2(s)} * \left\{\frac{\Lambda_{21}(s)}{f_1(s)} + a_5\frac{\Delta(s)}{(f_1(s))^2}\frac{e^{-\sqrt{\Delta(s)}}}{F_1(s)F_3(s)}\right\} + 1\right| \neq 0$$

on $Re(s=x+iy)$ positive.

The sufficient condition is

$$\left|\frac{4a_4}{a_2}\frac{(1 + e^{-(a_1-b_1)})^2}{(1 - e^{-(a_1-b_1)})^4} * \left\{\frac{1}{4\beta_3}\left(\frac{1}{(a_2 + a_3)} + \frac{\beta}{(a_1 - b_1)}\right)(1 - e^{-(a_1-b_1)})^2 + a_5e^{-(a_1-b_1)}\right\}\right| < 1$$

This condition results from a majoration of the function denominator modules. Notice that this condition depends on the hydraulic (F_r, χ) and geometrical (ks, kp) canal pool parameters and linearized gate discharge equation (c_{11},c_{12})

parameters. Because of the sufficient character of the stability condition, this one has to be evaluated in order to establish the stability of coupled subsystems.

Evaluation of the stability condition

The dimensionless number χ depends upon x_r, the reference canal pool length. As presented in its equation, for some geometric and hydraulic reference conditions, a great χ (respectively small) is associated to a long canal pool (respectively short canal pool). The evaluation condition is that the reference *Froude* number is less than one.

Notice that with respect to the unconditionallity of a subsystem stability, the two coupled subsystem dynamics can be unstable because of the (non linear) interaction of subsystems by way of the wave loops in the canal pools. In [4], it is proposed a test canal pool sorting. This classification is used in order to analyze the stability sufficient condition.

	$0<\chi<0.6$	...
$0<\eta<3$	*type 1. Non damped waves*	...
$\eta>3$	*type 3. Damped waves*	...
	$0.6<\chi<1.35$	$\chi>1.35$
$0<\eta<3$	*type 2. Non damped waves*	*
$\eta>3$	*type 4. Damped waves*	*type 5.Damped waves*

where $\eta=\chi/(F_r*(1-F_r))$ A great η value means that waves are damped [4]. It is considered that this type (*) of canal pool is not representative, naturally, of the irrigation canal pools. Canal pool clasification is constructed from the (η,χ) values. The used test canal pool characteristics are:

Type	L_h	*m*	*I*	*x*	*K*	Q_{mi}	Q_{ma}	y_{min}	y_{max}
				$F_r<0.5$					
1	*7*	*1.5*	*.0001*	*3000*	*50*	*3.5*	*14*	*0.97*	*2.12*
5	*8*	*1.5*	*.0008*	*6000*	*50*	*20*	*80*	*1.37*	*2.92*
				$F_r>0.5$					
1	*1*	*1.5*	*.001*	*550*	*60*	*3*	*12*	*0.98*	*1.82*
5	*1*	*1.5*	*.002*	*1200*	*71*	*0.5*	*2*	*0.30*	*0.63*

Notation is *I canal pool slope, x: canal pool length , K: inverse of Manning coefficient,* Q_{min}*: minimal discharge,* Q_{max}*: maximal discharge,* y_{min}*: minimalwater level and* y_{max}*: maximal water level.*

All test canal pool types were used in order to analyze the sufficient condition behavior [12]. In this paper, two types of test canal pool are selected (see tables) in order to show the behavior of the stability condition following the *Fr* number. In each case, they represent the extreme hidraulic behavior of the test canal pools.

Examples of stability condition evaluation

Some precisions have to be given before showing the evaluation results. The *Saint-Venant* equations have been linearized around a normal steady regime. For a normal regime, the transfer functions and the proper values depend on F_r, χ, *kp* and *ks* parameters.

These parameter values were calculated numerically for every pair (Q_n, y_n) by varying *0.01* the water level between the interval y_{min} and y_{max}. Canal pool dynamics is, in this sense established. Such a dynamics has to be coupled to gate dynamics. This latter one is also established from a linearization process by respecting the discharge continuity in the canal pool. In fact, it was made for the pair $(Q_{max}y_{max})$. To conserve c_{11} and c_{12} constants means to consider a same order variation of this parameters for other regime conditions. The initial angle corresponds to $\alpha_o=0.9\alpha_{max}$ $(\alpha_{max=45})$. The initial head was determined in order to satisfy this opening and the maximal discharge. The same steady regime was considered in both canal pools, so the second canal pool bottom was diminished in order to respect this head. Linearization gate parameters computation has been done before.

The used gate parameters c_{11} and c_{12} are presented for each figure and they are, of course, established for a dimensionless discharge equation. In the result presentation, the "*AMIL* stability condition" represents the sufficient stability condition evaluation. So, if this condition is less than one the coupled subsystem dynamics is stable, else such dynamics could be unstable.

For Figure 5, the gate parameters are : $c_{11}=35.0920$ and $c_{12}=19.4351$. The system could be unstable. The criterium reads in a non damped wave flow (η) and a great perturbation celerity ($F_r<<1$), the dynamics of two coupled subsystems could be unstable. Unstability increased if the canal pool length (χ) is short.

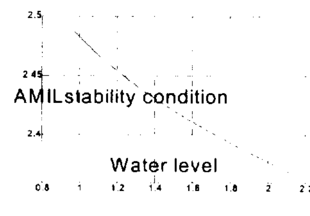

Figure 5 Evaluation of the criterium, type 1 with $F_r<0.5$

For Figure 6, the gate parameters are : $c_{11}=19.6751$ and $c_{12}=1.9543$. The system is stable. Criterium reads in a damped wave flow (η) and a weak perturbation celerity ($F_r<<1$), the dynamics of two coupled subsystems is stable. Stability augmented if the canal pool length (χ) is great. So, for a great χ, the evaluated criterium tendance is to *0*. This shows that the criterium allows to unconditional stability for two coupled long subsystems. It is verified in canal pool type *5)* with a $F_r<0.5$.

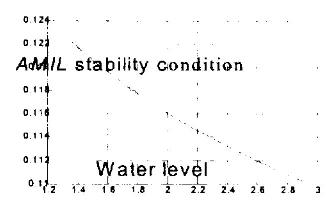

Figure 6 Evaluation of the criterium, type 5 with $F_r<0.5$

For figure 7, the gate parameters are $c_{11}=37.0468$ and $c_{12}=0.3619$. The system is stable. The criterium says in a non damped wave flow (η) and a weak perturbation celerity ($F_r<1$), the dynamics of two coupled subsystems is stable. Stability diminished if the canal pool length (χ) is short. The unstability tendence is present for a short canal pool (χ tends to *0*). This translates a close coupled subsystems.

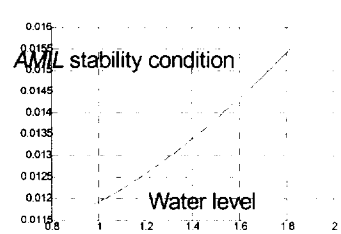

Figure 7 Evaluation of the criterium, type 1 with $F_r>0.5$

For Figure 8, the gate parameters are $c_{11}=24.8195$ and $c_{12}=0.4669$. The system is stable. The criterium says in a fort damped wave flow (η) and weak perturbation celerity ($F_r\text{->}1$), the dynamics of two coupled subsystems is stable. Stability established if the canal pools are strongly separated (criterium ~x.10^{-3}). For a great χ, the evaluated criterium tendence is to

O. This shows that criterium produces the unconditional stability for a two coupled long subsystem. It is verified in canal pool type *5)* with a $F_r>0.5$.

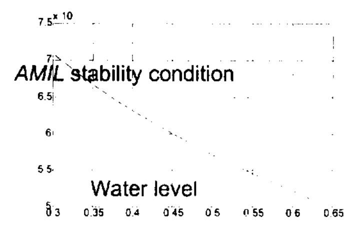

Figure 8 Evaluation of the criterium, type 5 with $F_r>0.5$

Now, it could be interesting to show the criterium utility. Let us search the minimal separation distance of the *AMIL* gates, in order to respect the stability of the coupled subsystems. Certainly, it is achieved by varying the reference length x_r of the canal pool. To illustrate this (see Figure 9), the canal pool characteristics type *1)* with $F_r>0.5$ are used. Even if this length varies, this canal pool is representative of this kind of test canal pools. We can say that such a system is stable for a minimal length, $x_r=86$ m.

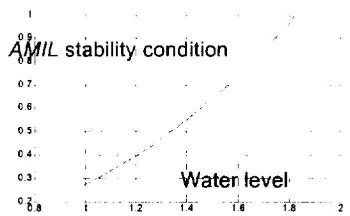

Figure 9 Evaluation of the criterium with a minimal x_r, type 1 with $F_r>0.5$

Discussion of results

In a subcritical flow with strong celerity waves ($F_r<<0.5$), the evaluated criterium is more sensible to the reference *Froude* number than the dimensionless length *(χ)*.

Behavior of the evaluated criterium with respect to the reference *Froude* number F_r. In subcritical regime conditions (small *Froude* number), the unstability problems are due to the dynamic waves (surface perturbations) and not to the mass waves *(χ)*. The dynamic waves produce the surface motion without real mass transfer. Stability criterium has showed that all the dynamic waves damp in a very long system. Physically translated, every surface (dynamic) wave is damped in a long distance even if the flow friction is weak. Results for $F_{r>0.5}$ have showed the same behavior.

According to the characteristics theory, this reference *Froude* number shows that the perturbation waves goes easily up to the canal pool if this number is small, and goes difficult up to the canal pool if this number increases. In fact, in supercritical flow, no waves goes up to the canal pool and there exists a real mass transfer.

Behavior of the evaluated criterium with respect to the dimensionless length χ. This criterium permits us to deduce the influence of close subsystems. This dimensionless number plays the role of a short or long distance. First, this criterium allows us to an unstable subsystem dynamics for every kind of test canal pool, when χ->*0*, *(a close subsystem)*. Second, the criterium tends to *0*, when χ is great *(a long separated subsystem)*. When determining the minimal gate distance, characteristics of a test canal pool type were no modified. Notice, however, that the computed distance does not take into account the gate dynamics. The criterium considers that gate reacts instantaneously (statically) to upstream local water level perturbations.

Brief, from the evaluated criterium study with respect to the coupled subsystem stability: the energy interaction between two subsystems is more important when down-upstream wave celerities are great and the interest of this stability sufficient condition is that it give us the conditions of a stable behavior in each test canal pool type.

7. CONCLUSIONS

Dynamics of coupled canal pool - *AMIL* gates has been studied. Function of the (canal pool) hydraulic (*Fr* and χ) and geometric (*ks* and *kp*) and the linearized gate discharge (c_{11} and c_{12}) characteristics, a set of parameters was found to be responsible of a stability criterium definition. First, from the subsytem stability study, it was established the unconditionallity of the subsystem stability and second for the coupled subsytem stability study, to satisfy this criterium means that transfer functions are analytical and the coupled subsystem dynamics is stable.

From the coupled subsytem stability study, result analysis has permitted to determine the criterium sensibility with respect to the reference *Froude* number and to sligth the interaction type between subsystems. A strong interaction is present when the perturbation celerity is great.

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