Optimal control of complex irrigation systems via decomposition-coordination and the use of augmented Lagrangian

H. El Fawal, D. Georges and G. Bornard
Laboratoire d'Automatique de Grenoble
INPG-CNRS
B.P. 46, F-38402 Saint Martin d'Hères Cedex, France

ABSTRACT

In this paper, we consider the optimal control problem for complex irrigation systems, using a receding horizon. The idea of decomposition is introduced for the goal of both reducing the computional complexity, to comply with the system topology and the monitoring architecture. A decomposition-coordination algorithm based on the use of both an augmented Lagrangian and the duplication of variables is developed which is suitable for the optimal control of complex irrigation systems, composed of water retention systems, water supply/distribution systems using canals and pipe networks. In some conventional decomposition-coordination approaches, such as the price decomposition-coordination algorithm, the coupling contraints between subsystems or the associated Lagrange multipliers are used as coordination variables. In our case, some physical variables, such as water flow rates, are duplicated in each subsystem, where they appear. Some compatibility constraints are then introduced and their associated Lagrange multipliers are used as coordination variables. In this paper, we present the application of this approach to to the Canal de la Bourne irrigation network, which irrigates the agricultural plain of Valence (South-East of France).

Keywords. Large scale systems, optimal control, decomposition and coordination methods, augmented Lagrangian, nonlinear programming.

1. INTRODUCTION

Most irrigation systems are based on a network of main, lateral (secondary), sublateral, tertiary, and quaternary canals. Water is taken from rivers or lakes and from boreholes (underground resources), and circulates in the water system through some canals and somtimes pipes, in order to be delivered finally to consumers. In this paper we will consider irrigation systems which combine both water retention systems, open-channel water supply systems and pipe distribution systems. The problems related to water supply network management ([1] and [2]), can be classified into three groups, according to the time horizon on which they are considered.

- Short-term problems: These problems arise in the daily management of the network.
- Mid-term problems: The middle-term problems are related to a more strategic management of the network, involving horizons longer than one day.
- Long-term problems: The long-terms problems concern essentially the optimal design of the network.

2. PROBLEM STATEMENT

In this paper we are concerned with short-term or mid-term planning problems of irrigation systems. Under the assumption that the values of user demands are known (on the basis of prediction methods [1], [7] for example) for a receding horizon T, the problem is to determinate at any time how to operate pumps, boosters, valves, gates and other control devices in order to minimize some cost function and to satisfy users demands. The operating cost of the network is composed on one hand of pumps, boosters and gates electric consumption expenses, and on the other hand of water waste costs. This objective function has to be minimized under the local and the network dynamic constraints. The local constraints arise from physical limitations and from operation considerations (minimal and maximal bounds of reservoir levels, maximal variation rates, maximal pipes flows, ...). Some of the constraints are strengthened in order to insure network security. In the case of water systems with electrical power stations (such as the Canal de la Bourne system) we have to substract

0-7803-4778-1/98 $10.00 © 1998 IEEE

some profits due to power generation from the cost function.

2.1 Modelling of a large irrigation system

As we mentioned before, in an irrigation system we can have two types of water systems, water retention and water supply/distribution systems. Water retention systems provide reserves for water supply and distribution systems. A retention reservoir has a variable inflow $Q_I(t)$ and outflow $Q_R(t)$ and is represented by the following dynamics:

$$\begin{aligned}\frac{dW(t)}{dt} &= Q_I(t) - Q_R(t)\\ Q_I(t) &= \sum_{j\in s} Q_{Oj}(t) - \sum_{j\in U} Q_{Uj}(t)\\ &\quad + \sum_{j\in Z} Q_{Zj}(t)\end{aligned} \qquad (1)$$

where Q_I is an outflow which is at the same time an input flow to a canal, $W(t)$ is the reservoir volume which depends on the reservoir water level, Q_{Oj} are output flows from the canals connected to the specified node, Q_{Uj} denote water consumptions, and Q_{Zj} are the node boundary inflows. In this paper we consider that each canal is represented by a second-order linear dynamics with natural pulsation ω_n, damping coefficient η, gain K and with a pure transportation delay T:

$$\ddot{Q}_O(t) = -2\eta\omega_n\dot{Q}_0(t) - \omega_n^2 Q_0(t) + K\omega_n^2 Q_I(t-T) \quad (2)$$

For the water retention systems and the canals, the variables of the model can be now classified as follows:

State vector $x_{rc}(t) = (Q_O(t), W(t))$
Control vector $u_{rc}(t) = (Q_U(t), Q_R(t))$
Disturbance vector $z_{rc}(t) = (Q_Z(t))$

Water supply/distribution systems are composed of pipes, control valves, pump stations, treatment works, reservoirs and water consumers. The elements are interconnected to produce a network composed of nodes (reservoirs and treatment works) and branches (pump stations, pipes, valves). The equations of the network ([1], [2]) express flow conservation at nodes (Kirchoff conservation law) and relations between head losses and heads for arcs, namely:

$$\sum_{i\in J_j} q_{ij} = d_j \qquad (3)$$

$$\pi_{ij} = h_i - h_j, \qquad (4)$$

where d_j is the consumption at node j and q_{ij} is the algebraic flow from node i to node j (i is an adjacent node to j, and the set of such nodes is denoted J_j). Variable h_i is the head at node i and π_{ij} is the head loss in the pipe connecting nodes i and j. There exists for each arc a relationship between flow and head loss. For a pipe, the corresponding equation derived from the Hazen-Williams formula ([1]) is

$$\pi_{ij} = R_{ij}.q_{ij}.|q_{ij}|, \qquad (5)$$

where R_{ij} is the resistance of the pipe connecting node i to j. For a valve, a similar expression holds. For a pump, the head increases is usually approximated by a parabolic function

$$\pi_{ij} = a_{ij}.q_{ij}^2 + b_{ij}.q_{ij}c_{ij}. \qquad (6)$$

Equations (4)-(7) constitute a system of nonlinear static equations describing the instantaneous equilibrium of the network. The dynamic equations are related to the storage capacities and the canals (see equations (1) and (3)). In particular, the water level in the reservoir is a particular head variable. For the water supply/distribution systems, the variables from the model can be classified as follows:

State vector $x_s(t) = (q(t), h(t))$
Control vector $u_s(k) = (v(t), n(t), s(t))$
Disturbance vector $z_s(k) = (d(t))$

where q and h are respectively the pipe flows and heads throughout the network, and v, n, s are respectively the degrees of valve opening, and the number and speeds of pumps in operation, and d is the demand vector. Finally, in the case of a combined water retention, supply and distribution system, after time-discretization one gets a discrete-time implicit singular model (due to the coupled static and dynamic equations):

$$E.x(k+1) = F(x(k), u(k), z(k)), \qquad (7)$$

where matrix E is singular, $x = (x_{rc}, x_s)$ is the state vector, $u = (u_{rc}, u_s)$ is the control vector, and $z = (z_{rc}, z_s)$ is the disturbance vector.

2.2 Formulation of the optimal planning problem

We seek to find the optimal control $u(k)$, $k = 0, ..., T-1$, where T represents the control horizon.

with time step of one hour, which minimize the following cost funtion:

$$J(x,u) = \sum_{k=t}^{t+T-1} (P(x(k),u(k)) + Pe(x(k),u(k)) - Pr(x(k),u(k))) \quad (8)$$

where

- $P(x,u)$ is the cost function due to the pumping stations which is the summation over all the pump energy consumptions;
- $Pe(x,u)$ is the penalty cost function for the throwing out flows (water waste);
- $Pr(x,u)$ is the profit function due to power generation;
- T is the receding horizon.

subject to:

1. The state representation of the system (dynamic and static parts):

$$E.x(k+1) = F(x(k),u(k)), \quad (9)$$

2. The bound constraints:

$$\underline{x} \leq x(t+k) \leq \overline{x}, k = 1, ..., T, \quad (10)$$
$$\underline{u} \leq u(t+k) \leq \overline{u}, k = 0, ..., T-1. \quad (11)$$

We can reduce the problem to solving

$$\min_{u \in U_f} J(u) \quad (12)$$

where $u = (u(t), ..., u(t+T-1))$, U_f is the feasible set previously defined, and $J(u)$ is the total cost function defined by (8).

3. SOLUTION BASED ON DECOMPOSITION - COORDINATION

The large structure of irrigation systems suggests the use of decomposition techniques to reduce the problem size. Our approach is based on the ideas of decomposition-coordination [3], the duplication of some variables [5] and the use of an augmented Lagrangian formulation [4].

Let us consider the following class of optimization problems:

$$\begin{array}{l} \min_{u \in U_f} J_1(u) + J_2(u) \\ \Theta_1(u) \leq 0 \\ \Theta_2(u) \leq 0 \end{array} \quad (13)$$

The duplication of variables leads to the new problem:

$$\begin{array}{l} \min_{u \in U_f, v \in V_f} J_1(u) + J_2(v) \\ \Theta_1(u) \leq 0 \\ \Theta_2(v) \leq 0 \\ u - v = 0 \end{array} \quad (14)$$

J_1 and J_2 are two functionals from $\mathbb{R}^n$ to $\mathbb{R}$, U_f nd V_f are some closed subsets of $\mathbb{R}^n$. Θ_1 (resp. Θ_2) is a mapping from $\mathbb{R}^n$ to $\mathbb{R}^{m_1}$ (resp. $\mathbb{R}^{m_2}$). The additional constraints $u = v$ are called "compatibility constraints". The two formulations are obviously equivalent.

In order to illustrate this idea in the case of irrigation systems, we consider a simple canal stretch connected to a pipe network:

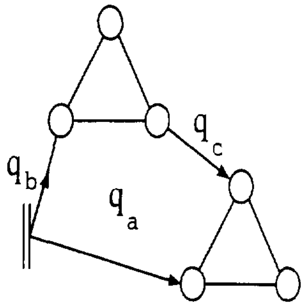

Figure 1: A simple network example

Let us consider the network of Fig. 1. In order to achieve the decomposition of this network into 3 subnetworks, the connections of the variables q_a, q_b, and q_c are fictitiously cut as shown in Fig. 2. This decomposition will create two independent flow vectors, the initial vector $q = [q_a \ q_b \ q_c]$, the so-called dual vector $\tilde{q} = [\tilde{q}_a \ \tilde{q}_b \ \tilde{q}_c]$, and three independent subnetworks.

More generally, we will consider now the decomposition into N subnetworks made of either canals or pipe

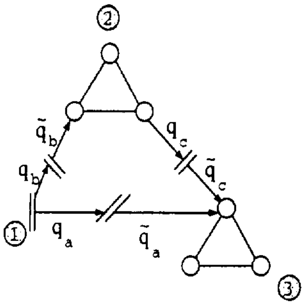

Figure 2: Network decomposition

subnetworks. After decomposition and duplication of variables, the problem may be formulated as follows:

$$\min_{u_i,\ q_i \in U_f^i,\ i=1,\ldots,N} \sum_{i=1}^{N} J_i(u_i,\ q_i) \tag{15}$$

subject to:

$$q_i - \sum_{j=1,j\neq i}^{N} E_{ij} q_j = 0, i = 1, \ldots, N \tag{16}$$

where u_i is the control vector of subnetwork i, q_i is the vector of flows interconnected with the other subnetworks, and U_f^i is the feasible set for each subnetwork i, $i = 1 \ldots N$. E_{ij} is a matrix whose entries are either 0 or 1. In the case of exemple of Fig. 2, $q_1 = (q_a, q_b)$, $q_2 = (\tilde{q}_b, q_c)$ and $q_3 = (\tilde{q}_a, q_c)$. The augmented Lagrangian associated to this problem is given by:

$$\begin{aligned} L(u_1, \ldots, u_N, q, p) &= \sum_{i=1}^{N} \{ J_i(u_i, q_i) \\ + < p_i, q_i - \sum_{j=1,j\neq i}^{N} E_{ij} q_j > &+ \frac{c}{2} \|q_i - \sum_{j=1,j\neq i}^{N} E_{ij} q_j\|^2 \} \end{aligned} \tag{17}$$

where p_i is the Lagrangian multiplier vector associated to each compatibility constraint. Since the quadratic term $\frac{c}{2}\ \|q_i - \sum_{j=1,j\neq i}^{N} E_{ij}q_j\|^2$ is not separable, and then $L(.)$ is not separable, classical decomposition-coordination methods cannot directly apply. In order to overcome this problem, an algorithm derived from Algorithm 14 in [4], is proposed, which uses a linearization of $\frac{c}{2}\ \|q_i - \sum_{j=1,j\neq i}^{N} E_{ij}q_j\|^2$.

For a network divided into N subnetworks, we will have to solve N independent subproblems, which are coordinated via a coordination level in order to guarantee that the comptability constraints are met, as shown below:

At iteration k, the following subproblems are solved simultaneously:

For each subnetwork (i=1, ...,N) apply step (1)

$$(1) \left[\begin{array}{c} \min_{u_i, q_i \in U_f^i} J_i(u_i, q_i) \\ - \sum_{j=1,j\neq i}^{N} < p_j^k + c(q_j^k - \sum_{i=1,i\neq j}^{N} E_{ji} q_i^k), E_{ji} q_i > \\ + < p_i^k + c(q_i^k - \sum_{j=1,j\neq i}^{N} E_{ij} q_j^k), q_i > + \frac{1}{2\epsilon} \|q_i - q_i^k\|^2 \\ \Rightarrow u_i^{k+1}, q_i^{k+1}, \ \epsilon > 0 \end{array} \right. \tag{18}$$

$$(2) \left[\begin{array}{c} p_i^{k+1} = p_i^k + \rho(q_i^{k+1} - \sum_{j=1,j\neq i}^{N} E_{ij} q_j^{k+1}), \\ i = 1, \ldots, N,\ \rho > 0 \end{array} \right. \tag{19}$$

$k \leftarrow k + 1$ and goto to step 1).

4. APPLICATION TO THE CANAL DE LA BOURNE SYSTEM

Our primary research goal is to automatically operate the "Canal de la Bourne" irrigation system, in order to improve water distribution efficiency and safety [6]. This irrigation system consists of a 45-kilometer long canal connected to two secondary canals (called S2 and S3) and two main reservoirs supplied by a small river (called "La Bourne"), representing more than 70 km of canals, as represented by Fig. 3. More than 20 pumping stations are distributed along the canals, bringing water to the agricultural plain of Valence (South-East of France).

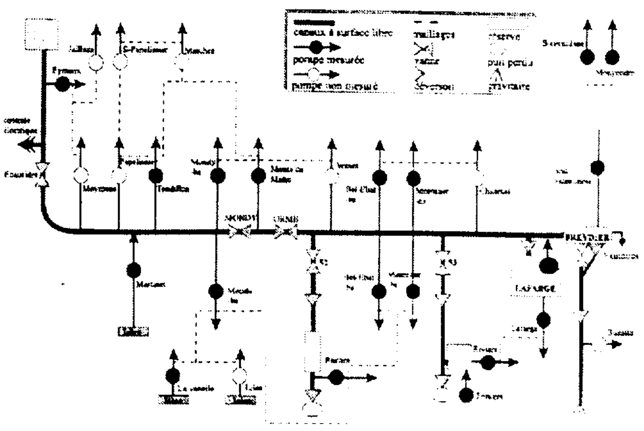

Figure 3: System structure

In order to illustrate the application of this algorithm on a simple example, we propose to formulate the short-term optimal control of the main section of this irrigation system in terms of water demands. This section begins at the regulator gate called "Orme" on the main canal and ends with the reservoir called "Freydier" and the pumping station called "Sud Valentinois". The following tables sum up the notations used for the control problem formulation:

Type	name	Water flow rate
Pumping	La Vanelle	i_1
Regulator gate	Orme	q_1
Regulator gate	S2	q_2
Regulator gate	S3	q_3
Pumping	Lafarge	i_2
Pumping	Riviers	i_3

Table 1: Control units

Name	Water flow rate, user demand
Mondy-hs	p_1, d_1
Mondy-bs	p_2, d_2
Monts du Matin	p_3, d_3
Ruches	p_4, d_4
Bel-Ebat-hs	p_5, d_5
Bel-Ebat-bs	p_6, d_6
Montelier-hs	p_7, d_7
Montelier-bs	$p_8 = d_8$
Riviers	p_9, d_9
Lafarge	p_{10}, d_{10}
Sud Valentinois	$p_{11} = d_{11}$

Table 2: Pumping stations

Name	Water volume
Orme	V_1
Ruches	V_2
Riviers	V_3
Lafarge	V_4
Freydier	V_5

Table 3: Reservoirs

The model of the system is given by the following equations:

Name	Water flow rate
Mondy-hs/Monts du Matin	$q_{1,3}$
Mondy-bs/Ruches	$q_{2,4}$
Ruches/Bel-Ebat-bs	$q_{4,8}$
Bel-Ebat-hs/Montelier-hs	$q_{5,7}$
Lafarge/Riviers	$q_{10,9}$

Table 4: Pipe connections

Kirchoff laws:

$$\begin{aligned}
d_1 &= p1 + q_{1,3}\\
d_2 &= i_1 + p_2 - q_{2,4}\\
d_3 &= p_3 - q_{1,3}\\
d_4 &= p_4 + q_{2,4} - q_{4,8}\\
d_5 &= p_5 - q_{5,7}\\
d_6 &= p_6 + q_{4,8}\\
d_7 &= p_7 + q_{5,7}\\
d_9 &= p_9 + q_{10,9}\\
d_{10} &= p_{10} - q_{10,9}
\end{aligned} \tag{20}$$

Main canal dynamics:

$$\begin{aligned}
q_5(k) &= q_1(k) - q_2(k)\\
q_6(k) &= F_1(q_6(-), q_3(-), p_5(-), p_6(-), p_7(-), p_8(-))\\
q_7(k) &= i_2(k) + q_6(k)
\end{aligned} \tag{21}$$

where $x(-)$ refers to past values of the variable x; q_1 is the water flow rate at the regulator gate called "Orme".

Secondary canal S2 dynamics:

$$q_2'(k) = F_2(q_2'(-), q_2(-)) \tag{22}$$

Secondary canal S3 dynamics:

$$q_3'(k) = F_3(q_3'(-), q_3(-)) \tag{23}$$

Reservoir dynamics:

$$\begin{aligned}
&V_1(k+1) = V_1(k)\\
&+(q_0(k) - q_1(k) - p_1 - p_2 - p_3)\\
&V_2(k+1) = V_2(k)\\
&+(q_2'(k) - p_4(k) - g_2(V_2(k)))\\
&V_3(k+1) = V_3(k)\\
&+(q_3'(k) - p_3(k) - p_9(k) - g_3(V_3(k)))\\
&V_4(k+1) = V_4(k)\\
&+(g_4(V_5(k)) - i_2(k) - p_{10}(k))\\
&V_5(k+1) = V_5(k)\\
&+(q_7(k) - g_5(V_5(k)) - p_{11}(k))
\end{aligned} \tag{24}$$

where the g_i's represent the algebraic model of the sweirs; q_0 is the upstream flow rate at the beginning of the main canal section.

The cost function J is expressed as the summation of the pumping cost J_1:

$$J_1 = \sum_{k=t}^{t+T-1} \{\sum_{j=1}^{3} P_j^i(i_j) + \sum_{j=1}^{11} P_j^s(p_j)\} \qquad (25)$$

where the $P_j^i(.)$'s and $P_j^s(.)$'s represent the pumping cost functions, and of the penalty costs for wasting water at the sweirs, denoted J_2:

$$J_2 = \sum_{k=t}^{t+T-1} \sum_{j=2}^{5} K_j g_j(V_j)^2, \; K_j > 0 \qquad (26)$$

All the variables involved in this problem are supposed to bounded. The related optimal control problem appears to be nonlinear and non convex.

In order to illustrate the here-proposed decomposition - coordination algorithm, we consider the case when the system is divided into two sections: the main canal + the secondary canal S3 + Lafarge reservoir and the secondary canal S2. In this case, variables $q_{2,4}$, $q_{4,8}$ and q_2 have to be duplicated ($\tilde{q}_{2,4}$, $\tilde{q}_{4,8}$, $\tilde{q}_2$). Therefore, three additional compatibility constraints have to be introduced:

$$q_{2,4} = \tilde{q}_{2,4}; \; q_{4,8} = \tilde{q}_{4,8}; \; q_2 = \tilde{q}_2 \qquad (27)$$

Finally, application of the here-proposed two-level algorithm leads to:

Main Canal: Iteration k

$$(1a) \left[\begin{array}{l} \min \sum_{k=t}^{t+T-1} \{\sum_{j=1}^{3} P_j^i(i_j) + \sum_{j=1,j\neq 4}^{11} P_j^s(p_j) \\ + \sum_{j=3}^{5} K_j g_j(V_j)^2\} \\ +(p_1^k + c(q_{2,4}^k - \tilde{q}_{2,4}^k))q_{2,4} \\ +(p_2^k + c(q_{4,8}^k - \tilde{q}_{4,8}^k))q_{4,8} \\ +(p_3^k + c(q_2^k - \tilde{q}_2^k))q_2 \\ +\frac{1}{2\epsilon}(q_{2,4} - q_{2,4}^k)^2 + \frac{1}{2\epsilon}(q_{4,8} - q_{4,8}^k)^2 + \frac{1}{2\epsilon}(q_2 - q_2^k)^2 \end{array} \right. \qquad (28)$$

s.t. dynamics of the main canal.

Secondary canal S2: Iteration k

$$(1b) \left[\begin{array}{l} \min \sum_{k=t}^{t+T-1} \{P_4^s(p_4) + K_2 g_2(V_2)^2\} \\ -(p_1^k + c(q_{2,4}^k - \tilde{q}_{2,4}^k))\tilde{q}_{2,4} \\ -(p_2^k + c(q_{4,8}^k - \tilde{q}_{4,8}^k))\tilde{q}_{4,8} \\ -(p_3^k + c(q_2^k - \tilde{q}_2^k))\tilde{q}_2 \\ +\frac{1}{2\epsilon}(\tilde{q}_{2,4} - \tilde{q}_{2,4}^k)^2 + \frac{1}{2\epsilon}(\tilde{q}_{4,8} - \tilde{q}_{4,8}^k)^2 + \frac{1}{2\epsilon}(\tilde{q}_2 - \tilde{q}_2^k)^2 \end{array} \right. \qquad (29)$$

s.t. dynamics of canal S2.

$$(2) \left[\begin{array}{c} p_1^{k+1} = p_1^k + \rho(q_{2,4} - \tilde{q}_{2,4}), \\ p_2^{k+1} = p_2^k + \rho(q_{4,8} - \tilde{q}_{4,8}), \\ p_3^{k+1} = p_3^k + \rho(q_2 - \tilde{q}_2), \\ \rho > 0 \end{array} \right. \qquad (30)$$

$k \leftarrow k+1$ and goto to levels (1a) and (1b). We can notice that the levels (1a) and (1b) can be solved in parallel.

5. CONCLUSIONS

In this paper, we have presented a new decomposition - coordination algorithm suitable for solving optimal control of large-scale systems. It is based on both the duplication of some variables and the use of an augmented Lagrangian formulation, in order to ensure existence of saddle-point in the non convex framework. To illustrate this approach, we have considered the optimal control of a real irrigation system.

References

[1] M.A. Bryds. *Water systems.* Prentice Hall, 1994.

[2] P. Carpentier and G. Cohen. Applied mathemetics in water supply network management. *Automatica*, 29(5):1215–1250, 1993.

[3] G. Cohen. *Décomposition et coordination en optimisation déterministe différentiable et non différentiable.* PhD thesis, Université de Paris Dauphine, 1984.

[4] G. Cohen and D. Zhu. Decomposition coordination methods in large scale optimization problems. *Advances in Large scale systems*, 1:203–266, 1984.

[5] D. Georges. Optimal unit commitment in simulations of hydrothermal power systems: An augmented lagrangian approach. *Simulation practice and theory*, 1:155–172, July 94.

[6] D. Georges and H. El Fawal. Modeling and identification of the canal de la bourne irrigation system. application to a predictive control strategy. *International workshop on regulation of irrigation canals*, April 97.

[7] M. Tomczak. *Modélisation ARMA de séries chronologiques. Application à l'automatisation d'un réseau d'irrigation.* PhD thesis, Université de Nancy I, Janvier 1990.