Decentralized volume control of open-channels using H_2 norm minimization

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ABSTRACT

A lumped-parameter model is considered for open-channel networks that expresses the dynamic relationship, in terms of state space variables, between gate opening sections and stored water volume variations in the different canal reaches with respect to a reference configuration of uniform flow. A procedure is suggested to approximate a real steady flow condition with an ideal uniform flow configuration. A decentralized control is obtained by determining the state feedback gain matrix, whose structure is imposed to be diagonal, that minimizes the H_2 norm of a suitable transfer matrix. A numerical validation of the linear model used to synthetize the control law is proposed. A comparison between the behaviour of the linear model and that of the non-linear unsteady one, whose evolution has been determined using the SIC software, is proposed.

1. INTRODUCTION

In the last decades, much research effort has been devoted to water flow control of open-channel hydraulic systems such as irrigation channels. A great number of regulation procedures have been proposed. A very detailed classification has been done by Malaterre in his Ph.D thesis [9]. These methods differ for the choice of: *controlled variables*: discharges [12]; water levels or water level variations: upstream, downstream or in a middle point of the reach; volumes or volume variations [3]; *measured variables*: generally water levels; *control variables*: gate openings or gate opening variations, discharge or discharge variations; *logic control*: feedback or feedforward; centralized [3] and decentralized [13]. Another source of difference among the above mentioned methods is the definition of the model used for the control syntesis. Malaterre in [9] partitioned the different models in two categories. Models of the first type are based on physical laws and the considered variables have a physical meaning. Models of the second type are based on a mathematical representation of the type *black box*. The first type of model is available for irrigation channels: the water behaviour in an open-channel can be described by the Saint-Venant equations. The second type of model is based on the identification of transfer functions between all the inputs and outputs considered. The only drawback of this procedure is that the state vector has no physical meaning as it is derived by putting the transfer functions in a canonical form.

In this paper we consider a linear model deduced from the Saint-Venant equations [3]. The *state variables* are the volume variations with respect to a reference configuration of uniform flow. The *control variables* are the variations of the gate opening sections with respect to the same reference configuration. The proposed control law is an example of the so-called *constant volume control* which is really efficient, expecially as far as the speed of response is concerned. In previous works the above cited model has been used to obtain a centralized control law whose feedback gain matrix has been computed by applying an LQR technique. In this paper we design a decentralized control law by minimizing the H_2 norm of a suitable transfer matrix. This procedure is the equivalent, in the frequency domain, of the LQR technique in the time domain. The advantage of such an approach is the possibility to establish in advance the gain matrix structure. In our case the choice of a diagonal gain matrix allows us to design a decentralized control law that maintains the stored volumes in the different reaches practically constant, even with variations in users withdrawls, by acting only on the upstream gate of the reach whose volume variation is detected.

In this paper we compare the behaviour of the linear model with that of a non-linear model whose evolution has been determined using the SIC software developed by Cemagref [10].

Before proceeding in the description of the above model, we want to mention the existence of other linear models deduced from the Saint-Venant equations. Continuous time models have been obtained by means of discretization in the space domain: in such a way the partial derivatives with respect to time are substituted with total derivatives. The two Saint-Venant equations are replaced by a set of differential equations whose number increases as Δx decreases. This method has been used by Balogun in his Ph.D the-

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sis [1]. Garcia in [7] demonstrated that this procedure is not always effective unless the discretization step is really small: this implies an excessive dimension of the state space vector. Discrete time models have been obtained by linearization of the Saint-Venant equations as well. The hyperbolic nature of these equations allows quite different types of solution: the method of characteristics, the method of finite differences, finite elements and finite volumes. The method of characteristics has been used by Garcia in [7] with good results in the case of a channel with regular geometry and with rapid but small variations in the hydraulic conditions. The discretization with finite differences can be realized with both an explicite [11] and an implicite scheme [9]. Geometrical characteristics of the canal can impose a small space step. When using an explicite scheme, this small time step will impose a small time step for numerical stability reasons. If the hydraulic conditions vary quickly this is not a problem since a small time step will also be required to correctly model the hydraulic phenomena occurring in the canal. But if it is not the case this constraint is superfluous to bare. The second case, used by Malaterre [9], is surely much more convenient since no stability condition must be verified; so its validity is surely more general. The only drawback of the latter model is its high order. Therefore the linear state-space model proposed in [4] seems to be a good trade-off between precision and simplicity: its order is equal to the number of reaches. Its validity is obviously limited to small perturbations but this is a peculiarity of every linear model deduced by linearization around an equilibrium state. Furthermore it is only valid in the low frequency range, but it is at these frequencies that the most important phenomena in open-channels occur.

The paper is structured as follows. In Section 2 we recall the fundamental steps involved in the deduction of the linear state space model and discuss how to determine the reference configuration. In Section 3 we provide the necessary background to design a control law by minimizing an H_2 norm. In Section 4 we discuss how it possible to apply the above technique to design a decentralized control law. In Section 5 we consider a real applicative example and compare the linear system behaviour with the non-linear one. In the Appendix we collect all the notation relative to the system under study.

2. LINEAR APPROXIMATE MODEL

Consider the system sketched in Figures 1 and 2, consisting of a channel of N reaches joined by $N+1$ gates, where the last gate is fixed and the others are controlled. Let us suppose that water is conveyed to the first reach from a reservoir with constant level and that the level downstream from the final reach is also constant. All the variables considered, apart from those that define the geometry, represent the variations with respect to a reference configuration, assumed to be of uniform flow in each reach. In particular, let $\boldsymbol{v} = [v_1, \cdots, v_i, \cdots, v_N]^T$, $\boldsymbol{\sigma} = [\sigma_1, \cdots, \sigma_i, \cdots, \sigma_N]^T$, $\boldsymbol{q}_C = [q_{C1}, \cdots, q_{Ci}, \cdots, q_{CN}]^T$, where v_i, σ_i and q_{Ci} are the stored volume variation in the ith reach, the variation in the ith gate opening section and the user flow variation at the ith reach lower end, respectively. The other variables of interest are reported in the Appendix.

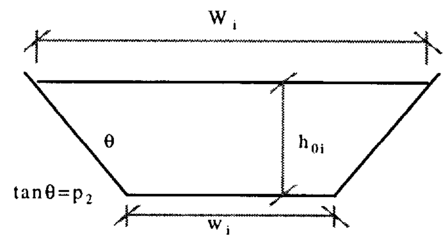

Figure 2: Trapezoidal canal cross section.

In general the initial configuration is of steady flow but not necessarily of uniform flow. In [4] an initial uniform flow condition has been assumed. In real applications, this is not the case. Therefore it is important to discuss how the steady flow configuration can be approximated with an ideal uniform flow condition. Since the model is in terms of volume variations, we have considered as a reference the uniform flow configuration characterized by the same volumes in each reach as those relative to the real steady condition. In such a way it is possible to obtain the constant water levels in each reach, so the uniform water profile is completely defined. Using the uniform flow equations it is easy to obtain the other reference conditions such as opening sections and discharges: obviously these last variables differ from the real ones.

The model derived in [4] and whose numerical validation is herein proposed, has been derived by first linearizing the Saint-Venant equations for the unsteady flow of water in open-channels [3] around a reference condition of uniform flow. Then, since the obtained equations are linear, the Laplace transform technique, with appropriate initial and boundary conditions, has been used to solve them. In such a way a model of the form

$$s\boldsymbol{V}(s) = \bar{\boldsymbol{A}}(s)\boldsymbol{V}(s) + \bar{\boldsymbol{B}}(s)\boldsymbol{\Sigma}(s)$$

can be obtained, where $\boldsymbol{V}(s)$ and $\boldsymbol{\Sigma}(s)$ are the Laplace transforms of $\boldsymbol{v}$ and $\boldsymbol{\sigma}$ respectively. $\bar{\boldsymbol{A}}(s)$ and $\bar{\boldsymbol{B}}(s)$ are $N \times N$ matrices of analytic functions. To obtain a linear and stationary approximate model, a Taylor series expansion of an appropriate matrix of analytic functions must be taken [4]. Since the model needs to be accurate in the low-frequency range, where the most significant phenomena take place, $s = j\omega = 0$ is taken as initial point and the series expansion may be truncated to the second term. Thus a model with the following structure can be obtained:

$$\dot{\boldsymbol{v}}(t) = \boldsymbol{A}\boldsymbol{v}(t) + \boldsymbol{B}\boldsymbol{\sigma}(t) \quad (1)$$

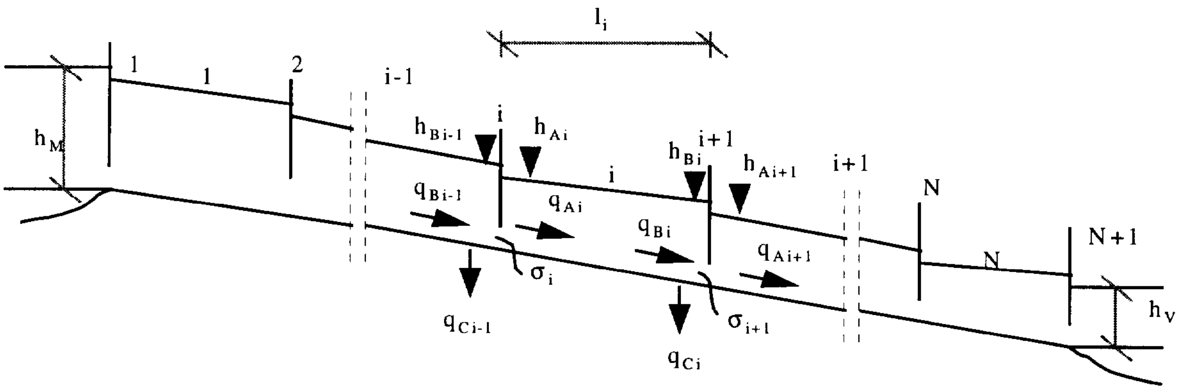

Figure 1: Scheme of system composed by a cascade of N canal reaches.

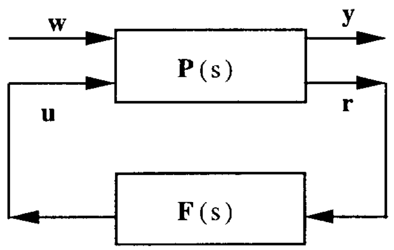

Figure 3: Linear fractional transformation scheme.

where $\boldsymbol{A}$ and $\boldsymbol{B}$ are constant matrices. Finally, taking into account the variations of the users flow rates $\boldsymbol{q}_C$, equation (1) can be rewritten as:

$$\dot{\boldsymbol{v}}(t) = \boldsymbol{A}\boldsymbol{v}(t) + \boldsymbol{B}\boldsymbol{\sigma}(t) - \boldsymbol{I}\boldsymbol{q}_C(t) \qquad (2)$$

where $\boldsymbol{I}$ is the N order identity matrix.

For more details on the construction of the mentioned linear model we address to [4].

3. CONDITIONS ON H_2 NORM

A standard approach to the control of linear time-invariant multiple-input multiple-output systems considers a block diagram such as the one shown in Figure 3 ([5] - [6]). In this figure $\boldsymbol{P}(s)$ is the transfer matrix of the generalized plant, while $\boldsymbol{F}(s)$ is the transfer matrix of the controller.

The vector $\boldsymbol{w}$ represents all external inputs, such as disturbances, sensor noise and reference signals, while the vector $\boldsymbol{y}$ is an error signal. The vector $\boldsymbol{r}$ is the set of observed variables used by the controller to compute the control input $\boldsymbol{u}$. The closed loop transfer matrix between $\boldsymbol{w}$ and $\boldsymbol{y}$ is called *lower linear fractional transformation* (LFT) of $\boldsymbol{P}$ and $\boldsymbol{F}$ and is denoted $F_l(\boldsymbol{P}, \boldsymbol{K})$.

Let us consider the linear model of a system to be controlled

$$\begin{cases} \dot{\boldsymbol{x}}(t) = \boldsymbol{A}\boldsymbol{x}(t) + \boldsymbol{B}\boldsymbol{u}(t) \\ \boldsymbol{z}(t) = \boldsymbol{C}\boldsymbol{x}(t) \end{cases} \qquad (3)$$

where $\boldsymbol{x} \in \mathbb{R}^n$ is the state vector, $\boldsymbol{u} \in \mathbb{R}^m$ is the control vector, and $\boldsymbol{z} \in \mathbb{R}^p$ is the ouput vector.

Classical LQR problem formulation [8] requires to find the state feedback law $\boldsymbol{u}(t) = \boldsymbol{K}\boldsymbol{x}(t)$, with $\boldsymbol{K} \in \mathbb{R}^{m \times n}$, such that the cost functional

$$J = \int_0^\infty [\boldsymbol{z}^T(t)\boldsymbol{Q}\boldsymbol{z}(t) + \boldsymbol{u}^T(t)\boldsymbol{R}\boldsymbol{u}(t)]dt \qquad (4)$$

is minimized for any initial state $\boldsymbol{x}(0) = \boldsymbol{x}_0$. Here $\boldsymbol{Q} = \boldsymbol{Q}^T \geq 0$, $\boldsymbol{R} = \boldsymbol{R}^T > 0$. The solution to the LQR problem is:

$$\boldsymbol{K}^* = -\boldsymbol{R}^{-1}\boldsymbol{B}^T\boldsymbol{X}, \qquad (5)$$

where $\boldsymbol{X}$ is the solution of the algebraic Riccati equation

$$\boldsymbol{X}\boldsymbol{A} + \boldsymbol{A}^T\boldsymbol{X} - \boldsymbol{X}\boldsymbol{B}\boldsymbol{R}^{-1}\boldsymbol{B}^T\boldsymbol{X} + \boldsymbol{C}^T\boldsymbol{Q}\boldsymbol{C} = 0.$$

The closed loop poles are the eigenvalues of $\boldsymbol{A}+\boldsymbol{B}\boldsymbol{K}^*$.

The equivalent frequency domain problem [6] is to find the state feedback matrix $\boldsymbol{K}^*$ such that the norm $\| F_l(\boldsymbol{P}, \boldsymbol{K}) \|_2$ is minimized [1], where the transfer matrix of the generalized plant $\boldsymbol{P}(s)$ has the following expression in terms of state space data:

$$\boldsymbol{P}(s) = \mathcal{C}(s\boldsymbol{I} - \mathcal{A})^{-1}\mathcal{B} + \mathcal{D} = \left[\begin{array}{c|c} \mathcal{A} & \mathcal{B} \\ \hline \mathcal{C} & \mathcal{D} \end{array}\right] =$$

$$\begin{array}{c} \\ \dot{\boldsymbol{x}}_P \\ \boldsymbol{y}_1 \\ \boldsymbol{y}_2 \\ \boldsymbol{r} \end{array} \begin{array}{c} \begin{array}{ccc} \boldsymbol{x}_P & \boldsymbol{w} & \boldsymbol{u} \end{array} \\ \left[\begin{array}{c|cc} \boldsymbol{A} & \boldsymbol{I} & \boldsymbol{B} \\ \hline \boldsymbol{Q}^{1/2}\boldsymbol{C} & 0 & 0 \\ 0 & 0 & \boldsymbol{R}^{1/2} \\ \boldsymbol{I} & 0 & 0 \end{array}\right] \end{array}.$$

The state space equation of the generalized plant is $\dot{\boldsymbol{x}}_p(t) = \boldsymbol{A}\boldsymbol{x}_p(t) + \boldsymbol{B}\boldsymbol{u}(t) + \boldsymbol{w}(t)$, i.e., it is the state

[1]Let $g(t) : \mathbb{R} \to \mathbb{R}^{m,n}$ be a signal matrix and $g(s)$ its Laplace transform. The H_2 norm of g is:

$$\| g \|_2 = \left(\int_{-\infty}^{\infty} trace\{g^T(t)g(t)\}dt\right)^{1/2} = \left(\frac{1}{2\pi}\int_{-\infty}^{\infty} trace\{g^H(j\omega)g(j\omega)\}d\omega\right)^{1/2}.$$

equation of the system (3) with an additional disturbance input $w(t)$. For a given $u(t)$, the evolution $x(t)$ of the system under arbitrary initial conditions $x(0) = x_0$ is the same of the evolution $x_p(t)$ of the generalized plant, initially at rest, when the external input is $w(t) = x_0\delta(t)$, where $\delta(t)$ is the Dirac impulse.

The closed loop output vector of the generalized plant is

$$y(t) = \begin{bmatrix} y_1(t) \\ y_2(t) \end{bmatrix} = \begin{bmatrix} Q^{1/2}Cx_p(t) \\ R^{1/2}u(t) \end{bmatrix} \quad (6)$$

and from (4) it can be seen that whenever $x(t) = x_p(t)$

$$\| y \|_2^2 = J. \quad (7)$$

Since

$$y(s) = F_l(P, K)w(s), \quad (8)$$

it is possible to prove [6] that the minimization of the norm $\| F_l(P, K) \|_2$ leads to a minimization of (7) for any external input of the form $w(t) = x_0\delta(t)$.

4. DECENTRALIZED CONTROL

We set our goal to that of designing a decentralized control law for an open-channel irrigation system whose model is of the form (1). The advantage of decentralized control is that central controller can be substituted by N local controllers (one for each reach). Each controller requires the measure of one reach volume and there is no need of transmitting informations to/from a central unit.

At this purpose, we impose a diagonal structure to the feedback gain matrix

$$K_d = diag\{k_1, k_2, \cdots, k_N\},$$

and we want to determine the K_d^* matrix whose control law is the best approximation of

$$u(t) = K^* x(t) \quad (9)$$

where K^* is the optimal (unconstrained) matrix. Therefore we want to determine the N parameters, $k_1, k_2, \cdots, k_N$ which minimize $\| F_l(P, K_d) \|_2$. Clearly, we will have that

$$\| F_l(P, K_d^*) \|_2 \geq \| F_l(P, K^*) \|_2 \quad (10)$$

since the RHS of equation (10) is a global minimum. The decentralized law performance will in general be worse than those given by (9).

Let us discuss the physical significance of H_2 norm minimization. We can say that if the generalized plant is excited with N different disturbances $w_i(t) = e_i\delta(t)$, where e_i is the ith canonical basis vector, and we call $y_i(t)$ the corresponding error signal, then

$$\sum_{i=1}^{N} \| y_i \|_2 = \| F_l(P, K_d) \|_2 \,. \quad (11)$$

Since the $\| y_i \|_2^2$ can be considered as the value $J_{d,i}$ of the performance index (4) when the decentralized system starts from the initial condition $x(0) = e_i$, the minimization of the $\| F_l(P, K_d) \|_2$ leads to the minimization of the $\sum_{i=1}^{N} J_{d,i}^{0.5}$ among all possible decentralized systems.

The decentralized system does not enjoy the fundamental property of optimal control, namely that of minimizing the performance index (4) for any initial state $x(0)$. It is possible, however to find upper bounds for the value J_d taken by (4) when the feedback matrix is K_d^*. Let $y(s)$ and $y_d(s)$ be the outputs of the generalized plant in the case of centralized and decentralized control when the input is $w(s) = x_0$. Then

$$\begin{aligned} y(s) &= F_l(P, K^*)x_0 \\ y_d(s) &= F_l(P, K_d^*)x_0. \end{aligned} \quad (12)$$

It is possible to prove [2] that the performance index of the centralized system is bounded by

$$J = \| y \|_2^2 \leq \| F_l(P, K^*) \|_2^2 \| x_0 \|_2^2, \quad (13)$$

while the performance index of the decentralized system is bounded by

$$J_d = \| y_d \|_2^2 \leq \| F_l(P, K_d^*) \|_2^2 \| x_0 \|_2^2 \,. \quad (14)$$

Note that in the previous equations $\| x_0 \|_2$ is the euclidean norm of a vector, while the norm $\| F_l(P, K_d^*) \|_2$ is a transfer function norm. These equations have a nice physical interpretation. They show that the value of the H_2 norm is an upper bound for the value of the performance index under arbitrary initial conditions on the unitary sphere. When the numerical values of $\| F_l(P, K^*) \|_2$ and $\| F_l(P, K_d^*) \|_2$ are close, we may conclude that for any arbitrary initial conditions the performance indexes J and J_d have close upper bounds.

Physically, the higher value of the performance index J_d is due to the fact that the decentralized system's response is slower. In fact, in the centralized control scheme each control input has immediate knowledge of the system's state, while in the decentralized control scheme this information is transmitted by the perturbation propagation.

5. APPLICATIVE EXAMPLE

The above described procedure has been applied to a two-reach canal, corresponding to the general scheme shown in Figures 1-2, with the following characteristics: length of the first reach: $l_1 = 4000$m; length of the second reach: $l_2 = 5000$m; canal bottom slope: $p_1 = 0.0003$; water level depth in upstream reservoir in reference to the canal bottom in the upper end section: $h_M = 2.5$m; water level depth in downstream reservoir in reference to the canal bottom in the lower end section: $h_V = 1$m; trapezoidal cross section (see Figure 2) with $w = 1.7$m, $\theta = 45°$; constant opening section of the third gate: $\sigma_3 = \sigma_{03} = 2.41\text{m}^2$. The reference configuration is characterized by the following discharge values: $q_{01} = 6\text{m}^3/\text{s}$, $q_{02} = 3.02\text{m}^3/\text{s}$, $q_{03} = 0.52\text{m}^3/\text{s}$, and by the following gate openings: $\sigma_{01} = 2.49\text{m}^2$, $\sigma_{02} = 1.58\text{m}^2$. The unknown disturbances are those reported in Figure 4.

The dynamic of this system can be represented by (1) where:

$$A = \begin{bmatrix} -0.1534 & 0.0556 \\ 0.1070 & -0.1902 \end{bmatrix} 10^{-3},$$

$$B = \begin{bmatrix} 1.8147 & -0.9691 \\ -0.2504 & 1.2601 \end{bmatrix}.$$

We have assumed that the output is equal to the state, so the C matrix in (3) is equal to an identity two order matrix. The weighting matrices are the same as those already used in preceeding works [3]:

$$Q = diag\{1, 1.22\}, \quad R = 50000\, diag\{1, 1\}.$$

To determine the optimal parameters we need to find the

$$\min_{k_1, k_2} \| F_l(P, K_d) \|_2 .$$

We used the software tools available in Matlab: `fmins` is the minimization procedure, and `normh2` computes the H_2 norm. The optimal values computed are: $k_1^* = -0.0070$, $k_2^* = -0.0060$. These values give $\| F_l(P, K_d^*) \|_2 = 366$. While the optimal K^* matrix is:

$$K^* = \begin{bmatrix} -0.0043 & -0.0011 \\ -0.0009 & -0.0047 \end{bmatrix}$$

and gives $\| F_l(P, K^*) \|_2 = 359$.

Note that finding the optimal decentralized control law is a problem of optimization. As such, it is almost certainly not a convex optimization problem and there may be multiple solutions that locally minimize the $\| F_l(P, K_d) \|_2^2$. However, in the case at hand starting from different initial values of k_1 and k_2 we observed that the minimization procedure always converges to the same value of K_d^*.

The results of simulation are reported in Figure 5: in a)-b) the volume percentage variations are shown, while the gate openings variations are reported in c)-d). It can be observed that there is not a perfect matching between the two sets of variables: it is not a surprising fact if all the simplifications in the model deduction are taken into account.

Different interesting structures can be considered: for example the diagonal K_d matrix can be substituted with a bandwidth matrix so each control variable is a function of more than one reach volume variation.

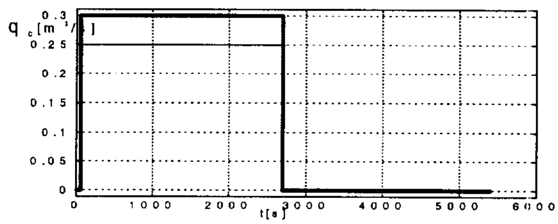

Figure 4: Unknown disturbances: q_{C1} (thick); q_{C2} (thin).

6. CONCLUSIONS

A lumped-parameter model for open-channel networks deduced by *Corriga et al.* in [4] has been examined. Volume variations in each reach with respect to a reference condition of uniform flow are assumed as state variables, while gate opening variations are assumed as control variables. In previous works the above cited model has been used to obtain a centralized control law whose feedback gain matrix has been computed by applying an LQR technique. In this paper a decentralized control law has been designed by minimizing the H_2 norm of a suitable transfer matrix ([5]-[6]). This procedure is the equivalent, in the frequency domain, of the LQR technique in the time domain. The advantage of such an approach is the possibility to establish in advance the gain matrix structure. In our case the choice of a diagonal gain matrix allows us to design a decentralized control law. Therefore, stored volumes in the different reaches remain practically constant, even with variations in users withdrawls, by acting only on the upstream gate of the reach whose volume variation is detected. The decentralized system does not enjoy the fundamental property of optimal control, namely that of minimizing the chosen performance index for any initial state. It is possible, however, to find upper bounds for the corresponding performance index when the feedback gain matrix is diagonal. Furthermore by numerical evaluation of two transfer matrices norms, it is possible to evaluate how much the performance index increases in the case of decentralization. A numerical validation of the model has been proposed by means of the commercial SIC software developed by Cemagref. The behaviour of the non-linear model has been compared with that of the linear one. Numerical results prove that differences are acceptable. Obviously the prediction capacity decreases as the flow configuration deviates from the reference one. The main advantages of the discussed modellization has been underlined in the introduction where a brief state of art of the problem is reported.

APPENDIX

Notation

h_M, h_V: constant water level depths in the up-

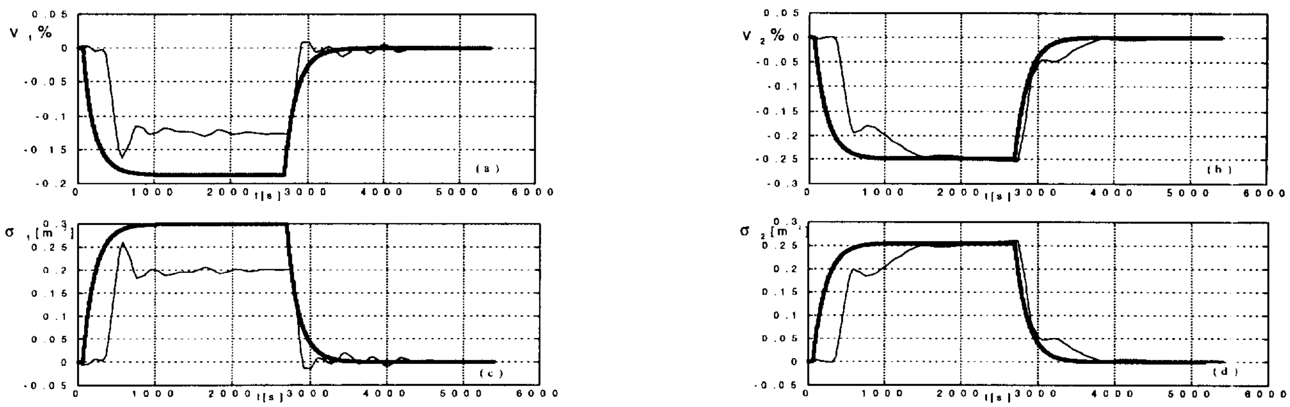

Figure 5: The results of simulation 1. (a) Evolution of percentage v_1 in the case of completely linear (thick) and completely non-linear model (thin). (b) Evolution of percentage v_2 in the case of completely linear (thick) and completely non-linear model (thin). (c) Evolution of σ_1 in the case of completely linear (thick) and completely non-linear model (thin). (d) Evolution of σ_2 in the case of completely linear (thick) and completely non-linear model (thin).

stream and downstram reservoirs, respectively;

h_{Ai}, h_{Bi}: upstream and downstream water level variations in the ith reach;

l_i: lenght of the ith reach;

σ_{0i}: opening section of the ith gate in reference condition;

σ_i: variation in the ith gate opening section;

N: total number of canal reaches;

p_1: canal bottom slope;

p_2: canal side slope;

q_{0i}: flow rate in the ith reach in reference condition;

q_{Ai}, q_{Bi}: upstream and downstream flow rate variations in the ith reach;

q_{Ci}: user flow variation at the ith reach lower end;

v_i: stored volume variation in the ith reach;

W_i: water surface width in the ith reach;

w_i: canal bottom width in the ith reach;

η: discharge coefficient.

References

[1] O. S. Balogun, "Design of real-time feedback control for canal systems using linear quadratic regulator theory ," *Ph.D Thesis*, Dep. of Mech. Eng. , University of California at Davis, 1985.

[2] G. Corriga, A. Giua, G. Usai, "An H_2 formulation for the desing of a passive vibration-isolation system for cars," *Vehicle System Dynamics*, Vol. 26, pp. 381–393, 1996.

[3] G. Corriga, S. Sanna, G. Usai, "Sub-optimal constant volume control for open-channel networks," *Appl. Math. Modelling*, pp. 262–267, July 1983.

[4] G. Corriga, S. Sanna, G. Usai, "Estimation of uncertainty in an open-channel network mathematical model," *Appl. Math. Modelling*, pp. 651–657, November 1989.

[5] J.C. Doyle, K. Glover, P.P. Kargonekar, B.A. Francis, "State space solution to standard H_2 and H_∞ control problem," *IEEE Trans. AC* , pp. 831–847, August 1989.

[6] R.L. Dailey, "Lecture notes for the workshop on H_∞ and μ methods for robust control," *30th IEEE Int. Conf. on Decision and Control*, Brighton, December 1991.

[7] A. Garcia, "Control and regulation of open channel flow," *Master of Science Thesis*, University of California, Davis, 1988.

[8] H. Kwakernaak, R. Sivan, *Linear Optimal Control Systems*, Wiley Interscience (New York), 1972.

[9] P.O. Malaterre, "Modelization, Analysis and LQR Optimal Control of an Irrigation Canal," *Ph.D Thesis*, LAAS-CNRS-ENGREF-Cemagref, Montpellier, 1994 (in French).

[10] P.O. Malaterre, J.P. Baume, "SIC 3.0, a simulation model for canal automation design," *Proc. Int. Workshop on Regulation of irrigation canals*, Marrakech, April 1997.

[11] J.M. Reddy, "Local optimal control of irrigation canals," *Journal of Irrigation and Drenage Engineering*, Vol. 116, No. 5, pp. 616–631, 1990.

[12] S. Sawadogo, "Modelization, predictive control and supervision of an irrigation system," *Ph.D Thesis*, LAAS-CNRS, Toulouse, 1992.

[13] J. Schuurmans, "Robust decentralized control of open-channels II: controller design," *Proc. Int. Workshop on Regulation of irrigation canals*, Marrakech, April 1997.