MIMO Predictive Control with Constraints by Using an Embedded Knowledge Based Model

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ABSTRACT

The study of hydraulic structures makes systematic use of mathematical models in order to verify their behaviour. On-line use of these models to synthesise predictive control permits basing control on almost perfect knowledge of every aspect of the process.
Achieving this aim requires good management of the embedded numeric model and the incorporation of an efficient resetting procedure. A simple method for identifying an adaptive linear model renewed at every step of the calculation permits applying the theoretical potential of PFC type predictive control.
The control can be calculated via an RST synthesis. This approach permits utilising the potential of the frequency study to validate the regulation's robustness and optimise its adjustments.

1. INTRODUCTION

The Compagnie Nationale du Rhône is a statutory operating company responsible for managing the Rhône in the areas of electricity generation, navigation, river maintenance and miscellaneous developments, as well as for flood management. Local control of river developments by predictive control has been developed in the framework of a joint EDF (Electricité De France) CNR (Compagnie Nationale du Rhône) project called "Rhône 2000" whose purpose is to renovate most of the automatic control devices installed on the Rhône. The co-ordinated control of all the local control devices used to facilitate the passage of floods is currently under study.

Predictive control is described in the literature in several ways. Clarke [1] proposes an approach called GPC (Generalised Predictive Control) while Sawadogo [2] uses it for dam-river systems and demonstrates its robustness with regard to variable delays. The formulation of our regulation is inspired by the PFC (Predictive Functional Control) method developed by Richalet [3]. Compas [4] has set-out the principle for different applications including the management of a hydraulic structure.

In the following chapters, we first present the modelling method used. The determination of predictive control via an RST synthesis is then described. Also, a method of optimising tuning by a frequency study is presented.
Lastly, the perspective of using MIMO predictive control for the co-ordinated control of a chain of structures is considered.

2. MODEL

The free surface flows are described by Barré de Saint-Venant equations:

$$L.\frac{\partial Z}{\partial t}+\frac{\partial Q}{\partial x}=q \quad \text{et} \quad \frac{\partial Q}{\partial t}+\frac{\partial\left(\beta.\frac{Q^2}{S}\right)}{\partial x}+g.S.\frac{\partial Z}{\partial x}+g.S.J_e=(u_q-\frac{Q}{S}).q$$

in which Q represents the flow of the river (m^3/s), Z distance across the basin (m), t the time (s), S the surface wetted perpendicular to the flow (m^2), L the width of the basin at section (m), J_e the slope of the energy line, q the run-off flow along the basins (m^2/s), u_q the speed of the run-off flow (m/s), β the speed distribution coefficient.
Their non-linear character as well as the complex river geometry makes it difficult to obtain directly a linear model capable of being used for an optional or predictive type control.

Since the beginning of the 1980s the CNR has been equipped with a simulator incorporating these discretised equations and using the finite differences method with a semi-implicit diagram known as a Preismann diagram (cf. Cunge [5]).
Developed in-house, this product known as CRUE has been improved over the years and has become indispensable both for operating the Rhône, for which the CNR is responsible and for its structure and canal engineering designs used around the world.

Two methods permit using CRUE as an embedded model for the predictive control of volume levels in canals and in river developments:

- The first consists in managing the control scenarios. This is described further on.
- The second consists in using CRUE in order to update an ARMA type linear model:

$$Z(n+1)=Z(n)+\sum_{j=1}^{Nc} a_j \cdot \delta Qc(n+1-j) + \sum_{k=1}^{P}\left(\sum_{j=1}^{Np^k} b_j^k \cdot \delta Qp^k(n+1-j)\right) \quad ❶$$

with P being the number of disturbance flows (water intakes for canals, tributaries for rivers), Nc: the size of the model corresponding to the command, Np^k: the size of the model corresponding to the disturbance k considered, $\delta Qp(n+1-j)=Qp(n+1-j)-Qp(n)$: the disturbance flow and

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$\delta Qc(n+1-j) = Qc(n+1-j) - Qc(n)$: the command flow, n being the present instant.

In both cases, it is necessary to know the state of the system at every step of the calculation. The mathematical model, which is constantly reset by measurements, automatically reconstructs the entire discretised flow line. A non-linear reconstructor of the state of the system is also obtained.

3. PRINCIPLE OF PFC PREDICTIVE CONTROL

General Description

PFC (Richalet [3]) is intended to determine the best control enabling the reduction of deviations between coincidence points located on a reference trajectory on horizon HC and the future level. This strategy is schematised in figure (2) in the appendix. The principles of this method are based on the three following elements :

⇒ Reference trajectory

A reference trajectory Z_{ref} is defined in the future to coincide with the set-point. Satisfaction is provided by an exponential type connection. The constant of this exponential - coefficient λ - regulates the response time in a closed loop and thus the dynamics. It is expressed by (with n being the present instant and $j = 0$ à H):

$$Z_{ref}(n+j) = Zc(n+j) - \lambda^{j}.(Zc(n) - Z(n))$$

⇒ Coincidence points

On this trajectory, particular points known as coincidence points are chosen at future times $n + h_s$ and are used as targets for the trajectory to be calculated by the model.

⇒ Structure of the future command

With the aim of simplifying the optimisation, and to guarantee the unity of the solution, it is necessary to structure the command law. The simplest choice is a polynomial structure:

$$\delta Qc(n+j) = \sum_{s=0}^{nb-1} \mu_s(n) \cdot Ub_s(j) = \mu(n)^T \cdot Ub(j)$$ with n as the present instant, $j = 0$ à H,

$$\mu(n)^T = \begin{pmatrix} \mu_0(n) & \mu_1(n) & \cdots & \mu_{nb-1}(n) \end{pmatrix} \text{ et}$$

$$Ub(j)^T = \begin{pmatrix} Ub_0(j) & Ub_1(j) & \cdots & Ub_{nb-1}(j) \end{pmatrix}$$

Calculation of the command by scenario management

A simple solution to obtain a predictive command of the first order is the application of an iterative management of command scenarios leading to progressive convergence.

In this case, a single coincidence point is chosen at a time $n + h_0$ with $h_0 = H$. The command structure is limited to a basic function (nb = 0), i.e. the basic unit step: $Ub_0(j) = 1 \Rightarrow \delta Qc(n+j) = \mu(n)$

Open loop simulations are carried out on the embedded model reset by applying flow command law scenarios. The outputs at the level of prediction horizon $H = h_0$ are recorded:

- scenario 1: command law: $\delta Qc_1(n+j) = 0$, $j = 0$ à $h_0 \Rightarrow$ free output: level $Z_1(n+h_0)$
- scenario 2: command law: $\delta Qc_2(n+j) =$ constant, $j = 0$ à $h_0 \Rightarrow$ forced output: level $Z_2(n+h_0)$

With the linearity scenario to the 1st order around an operating regime, the flow law is calculated by the relation:

$$\delta Qc(n) = \frac{Z_{ref}(n+h_0) - Z_1(n+h_0)}{Z_2(n+h_0) - Z_1(n+h_0)}.(\delta Qc_2(n) - \delta Qc_1(n))$$

which permits approaching coincidence point $y_{ref}(n + h_0)$ located on the reference trajectory. The command law sought is obtained via successive iterations and scenarios, and by using the secant method.

It is then applied for the step in progress. All these steps are renewed for each control time.

Taking the different constraints into account is easy:

- The constraints on the inputs (e.g., variation limit gradient, variation dead band) are applied directly to the result. The sliding horizon nature of this type of command then permits progressive convergence.
- The constraints on the outputs (e.g., complying with the drawdown, level limit gradient at any point of the reservoir) are managed by a command scenario management strategy.

4. RST SYNTHESIS OF THE PFC CONTROL

This type of approach was developed in 1996 by Boucher [6] with the Generalised Predictive Control (GPC). Once carried out, it enables us to carry out a frequency study of the predictive control.

Model and predictor

By totalling the expressions of relation ❶ of $Z(n+1)$ to $Z(n+i)$, we obtain:

$$\hat{Z}(n+i) = Z(n) + \sum_{j=1}^{Nc} a_j \left(\sum_{q=0}^{i-1} \delta Qc(n+1+q-j) \right) + \sum_{k=1}^{P} \left(\sum_{j=1}^{Np^k} b_j^k \left(\sum_{q=0}^{i-1} \delta Qp^k(n+1+q-j) \right) \right)$$

By developing, it is possible to separate the known part at instant n (1) and the part to be predicted (2), as a function of the relative values of i, Nc and the Np^k:

$$\hat{Z}(n+i) = \underbrace{Z(n) + \sum_{j=1}^{Nc-1} \alpha_{i,j} \cdot \delta Qc(n-j) + \sum_{k=1}^{P} \left(\sum_{j=1}^{Np^k-1} \beta_{i,j}^k \cdot \delta Qp^k(n-j) \right)}_{(1)}$$

$$\underbrace{+ \sum_{q=0}^{i-1} \varepsilon_{i,q+1} \cdot \delta \hat{Q}c(n+q) + \sum_{k=1}^{P} \left(\sum_{q=0}^{i-1} \varphi_{i,q+1}^k \cdot \delta \hat{Q}p^k(n+q) \right)}_{(2)}$$

with $$\alpha_{i,j} = \sum_{r=j+1}^{\min(Nc,i+j)} a_r \text{ , } \beta_{i,j}^k = \sum_{r=j+1}^{\min(Np^k,i+j)} b_r^k \text{ ,}$$

$$\varepsilon_{i,q+1} = \sum_{r=1}^{\min(Nc,i-q)} a_r \text{ et } \varphi_{i,q+1}^k = \sum_{r=1}^{\min(Np^k,i-q)} b_r^k \text{ .}$$

By carrying out a selective transformation in Z, the above expression is written as:

❷ $$\hat{Z}(n+i) = Z(n) + \alpha_i(z^{-1}) \cdot \delta Qc(n-1) + \sum_{k=1}^{P} \beta_i^k(z^{-1}) \cdot \delta Qp^k(n-1) + \sum_{q=0}^{i-1} \varepsilon_{i,q+1} \cdot \delta\hat{Q}c(n+q) + \sum_{k=1}^{P} \varphi_i^k(z) \cdot \delta\hat{Q}p^k(n)$$

with $$\alpha_i(z^{-1}) = \sum_{r=1}^{Nc-1} \alpha_{i,r} \cdot z^{-r+1}, \quad \beta_i^k(z^{-1}) = \sum_{r=1}^{Np^k-1} \beta_{i,r}^k \cdot z^{-r+1},$$

$$\text{et } \varphi_i^k(z) = \sum_{r=1}^{i} \varphi_{i,r}^k \cdot z^{r-1}$$

Structure of the control

The main particularity of the PFC predictive control is its choice of a structure for the control sought. Generally, a polynomial structure of nb basic functions is chosen:

$$\delta\hat{Q}c(n+q) = \sum_{s=0}^{nb-1} \mu_s(n) \cdot Ub_s(q) = \mu(n)^T \cdot Ub(q)$$

with $$\mu(n)^T = \begin{pmatrix}\mu_0(n) & \mu_1(n) & \cdots & \mu_{nb-1}(n)\end{pmatrix}$$

and $$Ub(q)^T = \begin{pmatrix}1 & q\cdot Te & \cdots & (q\cdot Te)^{nb-1}\end{pmatrix}$$

N.B: Te is the sampling period and Ub is composed of basic polynomials.

The third term of the expression ❷ above is thus written:

❸ $$\sum_{q=0}^{i-1} \varepsilon_{i,q+1} \cdot \delta\hat{Q}c(n+q) = \mu(n)^T \cdot Yb(i)$$

with

$$Yb(i)^T = \begin{pmatrix}\sum_{q=0}^{i-1} \varepsilon_{i,q+1} \cdot 1 & \sum_{q=0}^{i-1} \varepsilon_{i,q+1} \cdot q \cdot Te & \cdots & \sum_{q=0}^{i-1} \varepsilon_{i,q+1} \cdot (q\cdot Te)^{nb}\end{pmatrix}$$

PFC Criterion

The PFC predictive control criterion is written as:

$$J(n) = \sum_{i=1}^{nh} \left(Zref(n+h_i) - \hat{Z}(n+h_i)\right)^2$$

where $$Zref(n+h_i) = Zc(n+h_i) - \lambda^{h_i} \cdot \left(Zc(n) - Z(n)\right)$$

By injecting ❷ and ❸ in expression J(n) and by deriving, we obtain $\tilde{\mu}(n)$ which minimises this criterion.

$$\tilde{\mu}(n) = -\left(\Theta^T \cdot \Theta\right)^{-1} \cdot \Theta^T \cdot \Bigg[\begin{pmatrix}1-\lambda^{h_1}\\ \vdots \\ 1-\lambda^{h_{nh}}\end{pmatrix} \cdot Z(n) - \begin{pmatrix}z^{h_1}-\lambda^{h_1}\\ \vdots \\ z^{h_{nh}}-\lambda^{h_{nh}}\end{pmatrix} \cdot Zc(n)$$

$$+ \begin{pmatrix}\alpha_{h_1}(z^{-1})\\ \vdots \\ \alpha_{h_{nh}}(z^{-1})\end{pmatrix} \cdot \delta Qc(n-1) + \sum_{k=1}^{P} \begin{pmatrix}\beta_{h_1}^k(z^{-1})\\ \vdots \\ \beta_{h_{nh}}^k(z^{-1})\end{pmatrix} \cdot \delta Qp^k(n-1)$$

$$+ \sum_{k=1}^{P} \begin{pmatrix}\varphi_{h_1}^k(z)\\ \vdots \\ \varphi_{h_{nh}}^k(z)\end{pmatrix} \cdot \delta\hat{Q}p^k(n)\Bigg]$$

with $$\Theta = \begin{pmatrix}Yb(h_1) & \cdots & Yb(h_{nh})\end{pmatrix}$$

Only the first value of this sequence is applied to the system, the entire procedure being carried out again during the following sampling period Te according to the sliding horizon principle.

$$\delta Qc(n) = -\Sigma_1 \cdot \begin{pmatrix}1-\lambda^{h_1}\\ \vdots \\ 1-\lambda^{h_{nh}}\end{pmatrix} \cdot Z(n) + \Sigma_1 \cdot \begin{pmatrix}z^{h_1}-\lambda^{h_1}\\ \vdots \\ z^{h_{nh}}-\lambda^{h_{nh}}\end{pmatrix} \cdot Zc(n)$$

$$-\Sigma_1 \cdot z^{-1} \cdot \begin{pmatrix}\alpha_{h_1}(z^{-1})\\ \vdots \\ \alpha_{h_{nh}}(z^{-1})\end{pmatrix} \cdot \delta Qc(n) - \Sigma_1 \cdot z^{-1} \cdot \sum_{k=1}^{P} \begin{pmatrix}\beta_{h_1}^k(z^{-1})\\ \vdots \\ \beta_{h_{nh}}^k(z^{-1})\end{pmatrix} \cdot \delta Qp^k(n)$$

$$-\Sigma_1 \cdot \sum_{k=1}^{P} \begin{pmatrix}\varphi_{h_1}^k(z)\\ \vdots \\ \varphi_{h_{nh}}^k(z)\end{pmatrix} \cdot \delta\hat{Q}p^k(n)$$

with $$\Sigma_1 = \begin{pmatrix}1 & 0 & \cdots & 0\end{pmatrix} \cdot \left(\Theta^T \cdot \Theta\right)^{-1} \cdot \Theta^T$$

Finally, we obtain the RST formulation of the PFC predictive control schematised in figure (5) in the appendix..

$$S(z^{-1}) \cdot \delta Qc(n) = -R \cdot Z(n) + T(z) \cdot Zc(n) - \sum_{k=1}^{P} \left(z^{-1} \cdot U^k(z^{-1}) \cdot \delta Qp^k(n) + V^k(z) \cdot \delta Qp^k(n)\right)$$

5. TUNING

There are three tuning parameters to be determined and they have a relatively uncoupled action on the regulation characteristics.
Firstly, you must choose the number of basic functions nb (nb-1 order of the control structure), generally equal to the number of coincidence points (it must be less than or equal to nb to permit solving the problem). It only influences the precision of the regulator.

Then, it is necessary to choose the reference trajectory coefficient λ acting on the dynamics, and to a lesser degree on the robustness.
Finally, the position of the coincidence points influencing the robustness, and to a much lesser extent on the dynamics, must be determined.

Adjusting the regulator is often the most delicate stage in applied automation problems since it implies a certain number of compromises related to physical constraints and to the dual factor of dynamics/robustness inherent in all systems.
In our case the adaptive model makes this step even more complex. An adjustment compromise for different operating regimes must be found.

Let us consider that we must follow a set-point on a ramp whose future progression is unknown. To minimise the trailing error inevitable in this case, we must chose at minimum an order 1 control structure (step + ramp). We shall then have two coincidence points h_1 and h_2.
The transfer of the corrected open loop is written as:

$$T_{boc} = R \cdot \frac{z^{-1} \cdot A(z^{-1})}{(1-z^{-1}) \cdot S(z^{-1})}$$ where threshold R is a function of λ.

Given the shape of the Nyquist plots of T_{boc} for sections of natural rivers and canals (cf. The example of the curve in figure (4) in the

appendix), only the gain margin will be significant for qualifying the robustness of the regulation.
N .b.: This will not be valid unless the Nyquist plot passes to the right of the point (-1,0).
Since we want to qualify the coupling of the tuning parameters, the specific shape of T_{boc} allows us to write:

$$MG(\lambda, h_1, h_2) = \frac{R(\lambda = 0, h_1, h_2)}{R(\lambda, h_1, h_2)} \cdot MG(\lambda = 0, h_1, h_2)$$

Thus it will be possible to study $\frac{R(\lambda = 0, h_1, h_2)}{R(\lambda, h_1, h_2)}$ and $MG(\lambda = 0, h_1, h_2)$ independently to obtain adjustments guaranteeing the best dynamics/robustness compromise.

6. CONTROL OF A DEVELOPMENT ON THE RHONE

The development of the Rhône has various purposes (hydropower, navigation, water management, irrigation and leisure). For this reason, the river has been divided into sections most of which have been designed according to a standard architecture (cf. figure (1) in the appendix). A diversion of the riverbed comprises a hydropower plant and a wide gauge lock. The reservoir is created by a flood control dam on the reach of the by-passed river just upstream of the diversion..
Regulation is carried out by controlling an outflow (cf. figure (3) in the appendix), that of the hydropower plant if the flow is less than the maximum admissible flow of the plant (operation during power generation) or that of the dam if the flow is greater than the maximum admissible flow of the plant (level control during flooding).

The entire approach presented in the previous chapters is being developed and will be installed in the computers of 12 developments on the Rhône downstream of Lyon, in the framework of a joint EDF/CNR project called Rhône 2000. The role of regulation by predictive control described above will be to control reservoir levels during flooding by guaranteeing the safety of property and people along the river.

The first regulation software tests have been carried out for the development at Péage-de-Roussillon. The results obtained demonstrate the efficiency of the predictive control in comparison with a traditional PID control. The sensitivity of the adjustments, the model's adaptive character and the ease of integrating different constraints permit regulation that is dynamic, robust, and it considerably reduces wear on the control devices.

7. MIMO REGULATION OF SEVERAL RESERVOIRS OF THE RHONE DURING FLOODING

Optimising the management of the volumes along rivers during flooding has always given rise to concern by operators. Current procedures already permit the natural attenuation of floods but they remain, for reasons of primary safety, local to each development. A single, overall management procedure for all the developments can only improve this attenuation.
In order to implement this method, a system permitting the automatic control of a chain of developments from a central point during flooding must be designed. The difficulty of controlling a chain of reservoirs in a river consists in ranking and integrating a large number of contradictory constraints and objectives.
To test the feasibility of such an approach, we have developed a regulation system using MIMO predictive control, management of objectives and ranked constraints (cf. Pagès [7]).

To control several developments, we have built a MIMO model by aggregating linear models of developments on the Rhône.
The entire mathematical procedure for obtaining the control seen above is carried out using large sized variables that permit us to determine the control flow vector.
Adjustment optimisation and validation of the robustness of the MIMO predictive control could be obtained by determining singular values in the frequency domain by using an approach H^∞.

Besides the specific constraints of each development, overall management must limit the flows propagated along the river. To do this, we have assigned a useful volume to each reservoir expressed by a drawdown zone for the set-point level.
The main objective of centralised control is therefore to find the best set of controls, such that the levels to be regulated remain in their drawdown zones and that the peak of the flow propagated is attenuated as well as possible along the river. Naturally, the presence of non-measured disturbances and capricious tributaries makes this more difficult..

As safety must be guaranteed in the case of degradation (breaks in links and centralised control failures), we propose a structure based on local independent regulators but which are controlled by set-points that progress within the drawdown zone mentioned above. These set-points are formulated in real-time by a central station equipped with a global regulation system using MIMO predictive control then transmitted to each local regulator for application. This operation is schematised in figure (6) in the appendix.

8. CONCLUSION

By perfectly mastering the mathematical flow simulation model, software for local regulation comprising predictive control and an embedded mathematical model has been developed. The first series of simulation tests revealed excellent behaviour. Its installation at 12 sites accompanied with adapted computer tests is planned to take place in the next few months.

Overall co-ordinated control is envisaged in a second stage. The feasibility study summarised in the previous paragraph shows that it has good potential for reducing peak flood levels. The scope of application of this method is not restricted to managing the Rhône. Studies are being carried out on its application to a network of canals.

9. REFERENCES

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APPENDICES

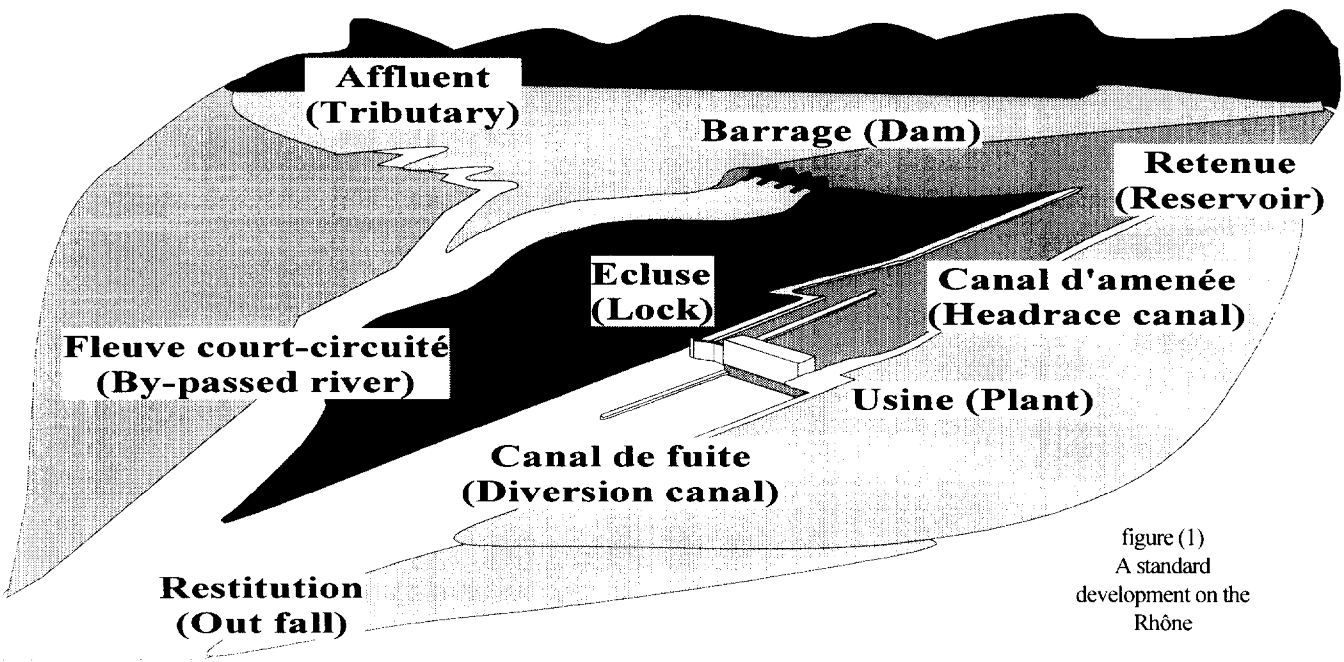

figure (1)
A standard development on the Rhône

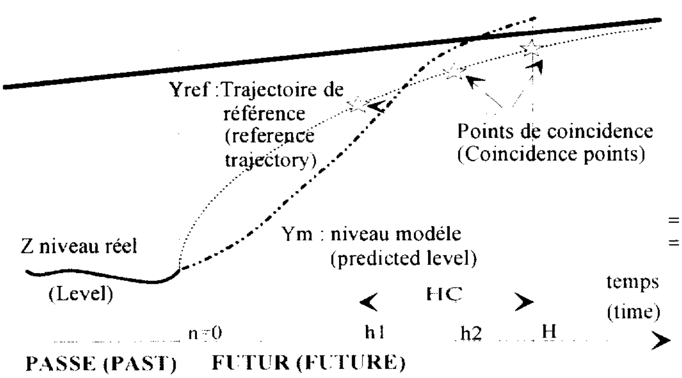

figure (2)
PFC predictive control

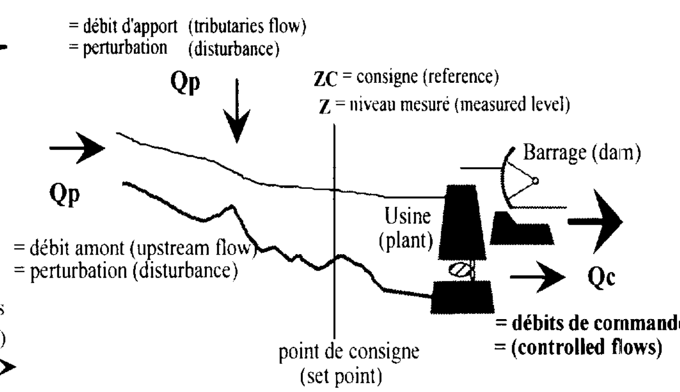

figure (3)
Control of a development on the Rhône

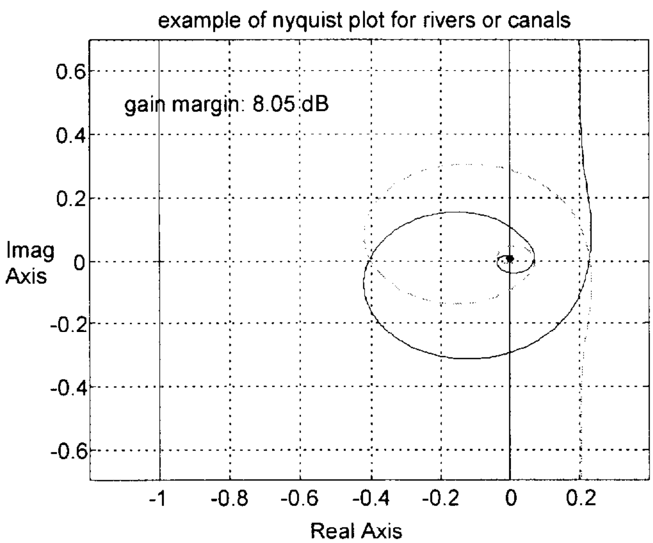

figure (4)
Nyquist curve of a corrected open loop transfer

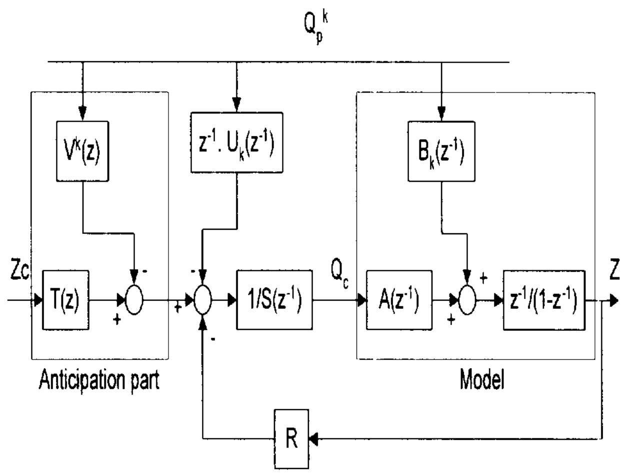

figure (5)
RST synthesis of the PFC predictive control

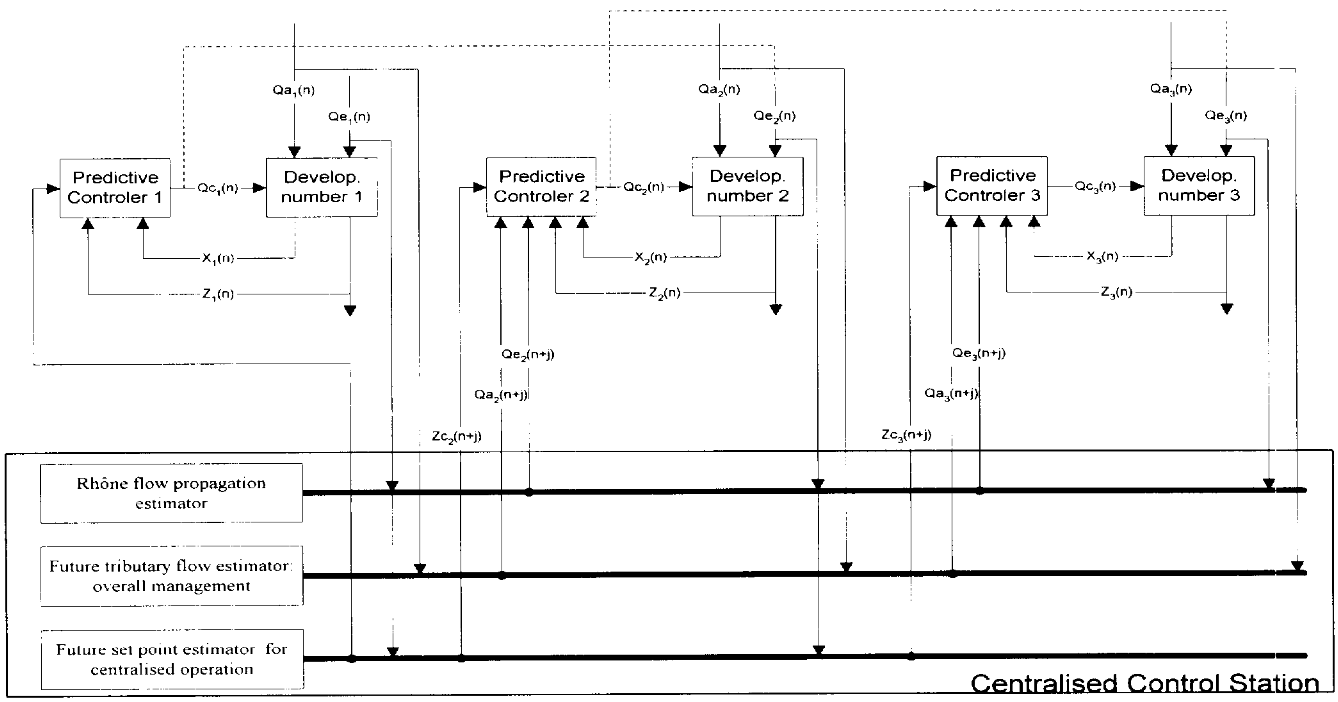

figure (6)
Centralised operation of local controllers - Qe: input flow - Qa: tributary flow
Qc: control flow - Z: real level - Zc: set point level - X: state vector