COMPARISON BETWEEN MONOVARIABLE CONTROLLER AND MULTIVARIABLE CONTROLLER

F. Sanfilippo
Engineering Service
Canal de Provence
13100 Aix-en-Provence France

J.L. Deltour
Engineering Service
Canal de Provence
13100 Aix-en-Provence France

ABSTRACT

This article describes the mathematical representation of two controllers which can be integrated in the Dynamic Regulation software closed loop module:

- a monovariable controller suitable for delay systems (Smith predictor) which is synthesized by a pole placement technique,
- a multivariable controller (control by feedback state) the purpose of which is to minimize a quadratic criterion. The multivariable representation chosen reduces the matricial calculations (dimension 5).

The behaviour of these controllers which regulate the downstream level of reaches using flows controlled gates are analyzed based on a simulation.

INTRODUCTION

On demand distribution of water requires the development of computer-aided operation tools which implement state-of-the-art-technology.

Automation of an irrigation canal with several reaches arranged in series has been studied for several years. Certain automatic systems are already applied, in particular on the canal de Provence with Dynamic Regulation. (Rogier et al 1987 [1], Coeuret 1977 [2], Deltour 1995 [3]).

However, in the Canal de Provence Company (SCP), research program is being persued in order to improve performances and operating safety on existing or future systems.

As part of this research, two controllers which can be integrated in a Dynamic Regulation closed loop module have been developed :

- a monovariable controller suitable for systems with delays,
- a multivariable controller.

This article describes the mathematical formulation to each controller. We compare their performances based on a simulation.

1. DYNAMIC REGULATION : OVERVIEW

Automatic and centralized Dynamic Regulation developed by SCP is designed to control the movements of water in conveyance system. It was first implemented in 1971 in answer to the technical constraints involved in operating open-channel flow.

1.1. Hydraulic constraints

With on-demand distribution of water, users can take the water as they need providing they don't exceed a maximum discharge. This leads to large flow variations which are difficult to predict and to program. Pressurized pipe networks are perfectly suited to this type of distribution. The only technical problem which has to be solved is economic management of buffer reservoirs and pumping stations.

In the case of canals the problems are not the same since these systems introduce substantial response times into the water transport system and offer little flexibility in the absence of on-line buffer volumes (reach reservoirs).

The regulation of canal flows is therefore largely influenced by the delay involved in the hydraulic propagation of the modification of a head discharge and the availability of the new discharge at a given point on the canal. To take account of this constraint, the regulation system set up on the canal can be based on two complementary approaches :

- The open loop which consists in making a forecast of variations in demand. The adjustments needed to satisfy future demand can be anticipated.
- The closed loop which controls the errors in the forecast or the adjustments at each regulation time step.

1.2. Basic principles of Dynamic Regulation

La régulation dynamique est un couplage entre la boucle ouverte et la boucle fermée. At the same time, it implements anticipatory adjustments based on predictions of consumption and continuous correction in order to adapt the state of the canal to real demand.

Dynamic Regulation software executes the following different tasks in real time :

- acquisition of measurements every 60 seconds through a remote control system,
- strict checks on validity and consistency of the incoming data, both in their relativity one to another, and in time (variations),
- storage of data in a real-time data base,
- estimated forecasting of requirements at main offtakes in order to anticipate variations in water demand. This short term forecast is executed and updated automatically on the basis of an analysis of past acquired data. It can be determined by the operator in the case of certain industrial users when consumption is based on a strict schedule,
- comparison of the canal status with a set status,
- determining of the canal facility manoeuvres from forecasting and canal status evaluation,
- verification of their validity,
- dispatch of control orders,
- checking of task execution.

2. IMPROVEMENT OF THE CONTROLLER

2.1. Introduction

For the last twenty years, progress in industrial control technology has brought improvements to the reliability and performances of automatic systems.
At the same time as the computer models for canal simulation were being developed, automation, the science of designing and executing automatic control systems, underwent a profound change and abandoned analog methods in favour of digital technology, more suited to industrial computer technology.

In order to benefit from the advent of digital technology, SCP engineers initiated a vast research programme associating the expertises of hydraulic engineers and automation engineers with a view to optimising the performances of closed loop regulation while retaining the logic behind the open loop/closed loop couple which so characterizes Dynamic Regulation. Two controllers are presently under study :

- A monovariable controller adapted to delayed (Smith Predictor) which was the subject of a thesis presented to Greenbelt Polytechnique in 1992 (Deltour 1992 [4]).
- A multivariable controller which was the subject of a thesis presented to University Lyon 1 in 1997 (Sanfilippo 1997 [5]).

2.2. Mathematical model of system dynamics

2.2.1. Purpose

Obtaining a reliable representative model constitutes one of the essential prerequisites for implementing an effective regulation diagram. Closed loop regulation is an automatic system which, through the use of control variables (discharges or gates opening), controls measured values (volumes, levels, flows).

Two sub-systems are involved in this type of operation (see figure 1) :

- the process to be controlled (the canal),
- the controller which calculates the adjustments needed to cancel the discrepancy between the measurements and the settings.

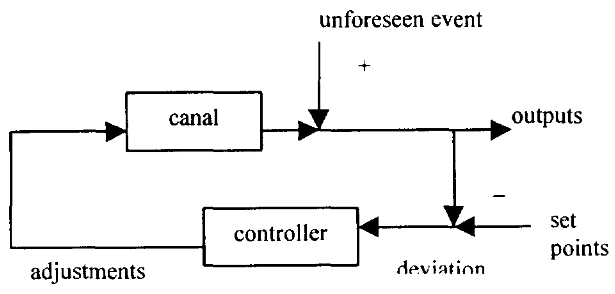


Figure 1: Basic principle of the closed loop

2.2.2. Choice of the input and output variables

An irrigation canal operates correctly if it delivers the contracted discharges to turnouts. These discharges are controlled through controlling the levels at strategic points on the canal.

An open canal operates in subcritical flow. Any disturbance observed downstream from a reach will have an impact upstream. It is therefore logical to control the downstream level of the reaches which will be the variables of the controller outputs.

The discharge/gate opening hydraulic relationships are strongly non-linear. To simplify the expression of the representation model, we propose to use discharges which are to be adjusted like control variables.

2.2.3. Description of the representative model

The representative model must be as simple as possible while faithfully describing canal behaviour. Two main dynamics describe the transient behaviour of a reach.

- The **Upstream discharge/Downstream level** dynamic : An analysis of the behaviour of the different canals has led us to choose the following transfer function (Deltour 1992 [4], Sanfilippo 1993 [6]) :

$$FT=\frac{G_{11}Z^{-r}}{1-Z^{-1}}+\frac{G_{21}Z^{-r}}{1-D_2Z^{-1}} \qquad (1)$$

The behaviour of the canal is then defined by a pure delay, one integrator pole, one pole (D_2) and two gains (G_{11} and G_{21}).

- The **Downstream discharge/Downstream level** dynamic which is modelled by the transfer function :

$$FT=\frac{G_{12}Z^{-1}}{1-Z^{-1}}+\frac{G_{22}Z^{-1}}{1-D_3Z^{-1}} \qquad (2)$$

consisting of two gains (G_{12}, G_{22}), one integrator pole and one pole (D_3). Contrary to the previous dynamic, the effects of a downstream discharge variation on the changes to the measurement are immediate.

2.3. Monovariable controller : Smith Predictor

2.3.1. Control logic

Using the upstream discharge of the reach, this monovariable controller must regulate the downstream level of the reach.
On a system which envolves a delay, a discrepancy between the measurement and the set point, may result from two causes :

- from a disturbance occurring since the last setting or which has not yet been entirely corrected. It is then necessary to adapt the setting as quickly as possible in order to correct this disturbance.
- from a former disturbance which has already been corrected but the effects of the correction are not yet visible due to the canal propagation time. A new modification to the setting would therefore result in an excessive correction.

This is why conventional controllers (Proportional Integral) cannot effectively correct phenomena which involve delays.

For these reasons we propose to use a regulator which is adapted to systems with delays (Smith predictor). As opposed to these conventional controllers, the Smith Predictor incorporates through internal model the predicted evolution of the measurement caused by changing the head discharge of the reach.
This enables the effects of adjustments made at previous control time steps to be taken into account without waiting for the impact on the measurement. Thus, the two possible origins of a discrepancy are separated and the performances of the controller improved.

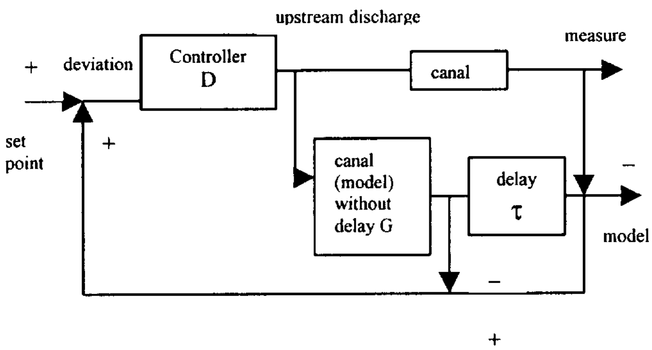


Figure 2 : Smith Predictor

2.3.2. Synthesis of the controller

This synthesis is the phase during which the type of the controller which will be used in the local control module is chosen.
If G (see transfer function (1)) is the representative model without delay and D the controller (Fig 2), the closed loop transfer function between the set point and the measurement is given by the following formula :

$$\frac{DG}{1+DG} \qquad (3)$$

We have chosen the following expression for the controller to ensure the stability and the accuracy of the control system :

$$D=\frac{(1+D_2Z^{-1})K_D(1-A_DZ^{-1})}{(1-Z^{-1})(1-Z_0Z^{-1})} \qquad (4)$$

- ✓ A_D and K_D are the coefficients to be determined which define the controller,
- ✓ Z_0 is the stable zero of the transfer function (1),
- ✓ D_2 is the transfer function (1) pole.

The pole placement technique is then used to determine the coefficients of setting (A_D and K_D).

2.4. Multivariable controller : Optimal control

2.4.1. Représentation multivariable d'un bief

Changes in the level downstream of a reach are subjected to two principal dynamics :
- the upstream discharge/downstream level monovariable dynamic which is represented by the transfer function (1),
- the downstream discharge/downstream level monovariable dynamic which is represented by the transfer function (2).

Implementing multivariable control requires modelling of the process dynamics to be controlled in the space state. Writing the above transfer functions in the space state by using an order 2 Padé approximater (Baranger (1991) [7]) leads us to the multivariable representation as per Sanfilippo (thesis 1997), Sanfilippo 1997 [5]).

$$\frac{dY}{dt}=AX+Bu$$

$$Y=CX$$

X : *state vector for dimension n*

U : *vector of commands for dimension p*

Y : *vector of measurements for dimension m*

$$A=\begin{Bmatrix} 0 & & & & \\ & -\frac{1}{T_1} & & & \\ & & \frac{-3}{\tau} & -\frac{\sqrt{3}}{\tau} & \\ & & \frac{\sqrt{3}}{\tau} & \frac{-3}{\tau} & \\ & & & & -\frac{1}{T_2} \end{Bmatrix} \qquad (5)$$

$$B=\begin{Bmatrix} G_{11} & G_{12} \\ G_{21}\ e^{-\frac{\tau}{T_1}} & 0 \\ G_{11}(b(0)+\bar{b}(0))+G_{21}(b(a)+\bar{b}(a)) & 0 \\ i(G_{11}(\bar{b}(0)-b(0))+G_{21}(\bar{b}(a)-b(a))) & 0 \\ 0 & G_{22} \end{Bmatrix}$$

$$C=\begin{bmatrix}1 & 1 & 1 & 0 & 1\end{bmatrix}$$

$T_1 =$ time constant of the Upstream discharge/downstream level dynamic

$T_2 =$ time constant of the Downstream discharge/downstream level dynamic

$$b(a)=\frac{1}{-3+i\sqrt{3}+a}\;\frac{3-i\sqrt{3}}{i\frac{\sqrt{3}}{6}}$$

A : Dynamic dimension matrix (n x n) which characterizes the evolution of the system in the absence of control.

B : Dimension command matrix (n x p) which determines how the control acts on the space system.

C : Matrix of controlled dimension outputs (m x n) which determines the measurements base on the state.

2.4.2. Control logic

Minimizing a quadratic criterion constitutes one of the means for determining a control for multidimensional linear systems. For very many physical systems, notably hydraulic processes, a quadratic criterion enables suitable expression of the overall qualities sought for the control. Indeed, these can be summarized by determining a control which constitutes the best compromise between performances (reduction of deviations) and the reduction of the adjustment setting modifications. The appearance of control variables in the expression of quadratic criterion to be minimized can be translated by the desire to limit the adjustment variations in order to obtain the required performances on measurements.

2.4.3. Controller synthesis : control by feedback state

While underlining, as previously, that the method for minimizing a quadratic criterion corresponds to a physical reality, the quadratic criterion also constitutes a mathematical tool, a calculation method for reaching the required form of control. The various weighting coefficients arising in the criterion are therefore no longer physical data linked to the canal but can be assimilated to adjustment parameters for defining performances of the control structure.

Let us consider the following criterion to be minimized :

$$J=\sum_{i=0}^{\infty}(u(i)^T Ru(i)+e(i)^T Qe(i) \qquad (6)$$

- e(i) = z - y(i) , z being the vector of the settings imposed at the outputs (level downstream reaches) y(i) of the canal.
- u(i) : the vector of controls which comprises all the discharges to be adjusted at gates.
- R, weighting matrix on the controls.
- Q, weighting matrix on the errors.

In other terms, the problem to be solved is : what are the adjustments to be made on the gates to keep the measurement equal to the setting while limiting the control variation range (to limit fatigue on control devices).

The expression of the control which enables the problem to be solved under constraint (6) is set out as follows (Foulard et al 1987 [8]; Borne et al 1993 [9]; Borne et al 1990 [10]; De Larminat 1993 [11]) :

$$u=-LX \qquad (7)$$

This gives a structure which is comparable to the traditional closed loops of the conventional monovariable controls.

L : gain matrix which makes it possible to take account of the space state and therefore to react in consequence in order to obtain the required behaviour. L is the solution to the following equation :

$$L=(R+B^T KB)^{-1}B^T KA \qquad (8)$$

where K is the asymptotic solution of the Riccati iterative equation :

$$K=A^T KA-A^T KB(R+B^T KB)^{-1}B^T KA+C^T QC \qquad (9)$$

2.4.4. Construction of the state observer

The reaction of the control supposes that the state vector is available i.e. physically measurable, which is generally not the case. To execute the control presented in the previous chapter (control by feedback state), we have made the synthesis of a state observer. This synthesis will make it possible to obtain the state of the system based on process inputs and outputs.

An observer is a linear system whose Xob state is such that (Foulard et al 1987) :

$$X^{+}{}_{0b}=AX_{0b}+Bu+G(Y-Y_{0b}) \qquad (10)$$
$$Y_{0b}=C(AX_{0b}+Bu)$$

Y_{0b} represents the best estimate which can be obtained for the output based on the observed state.

G is a solution gain matrix of :

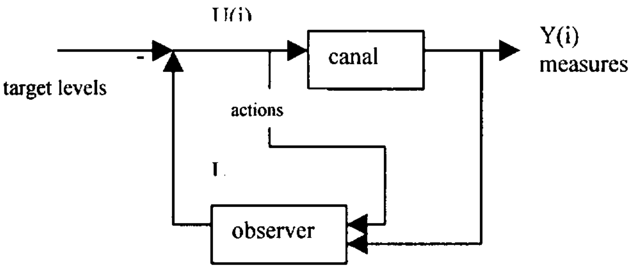
$$G^T=(R_1+CAK_1A^TC^T)^{-1}CAK_1A^T \qquad (12)$$

and K1, solution of the following Riccati equation :

$$K_1=AK_1(A^T-A^TC^T(R_1+CAK_1A^TC^T)^{-1}CAK_1A^T)+Q_1 \qquad (13)$$

R_1 and Q_1 are the weighting matrixes.

The form of the multivariable controller for controlling the level downstream of reaches using discharges to be adjusted at the gates can be set out diagrammatically as follows :



3. ANALYSE DES PERFORMANCES

Les performances des deux controller présentés ci-dessus ont été analysées en simulation sur trois biefs du canal de Marseille Nord (Canal de Provence extension du canal du Verdon 1977).

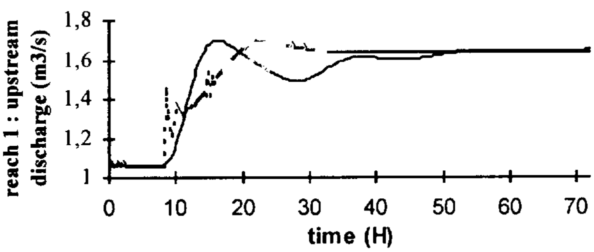
Table 1 : description des biefs

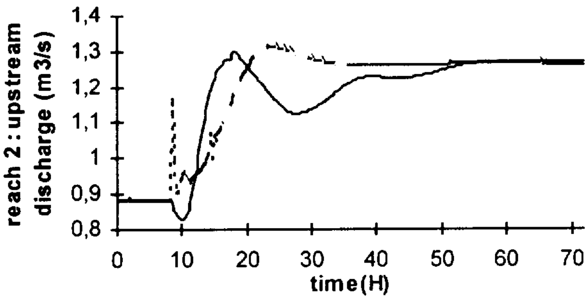
Bief 1	Bief 2	Bief 3
Longueur : 4400 m Pente : 1×10^{-3}	Longueur : 6500 m Pente : 9×10^{-4}	Longueur : 3000 m Pente : 8×10^{-4}

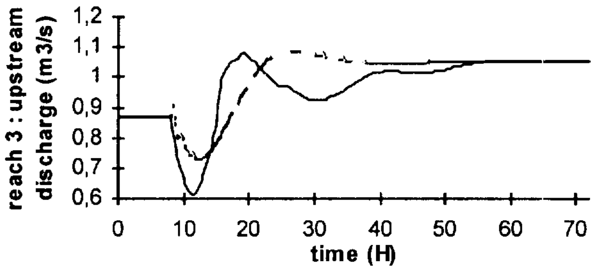
Le scénario de test est présenté dans la table ci après :

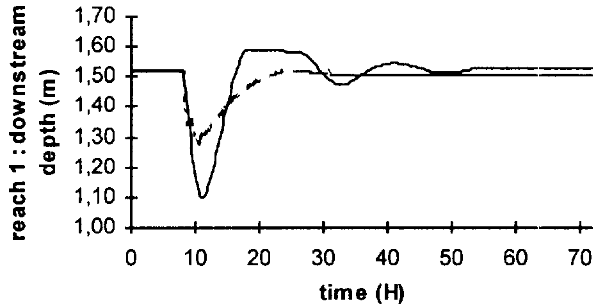
Table 2 : Studied simulation scenarios

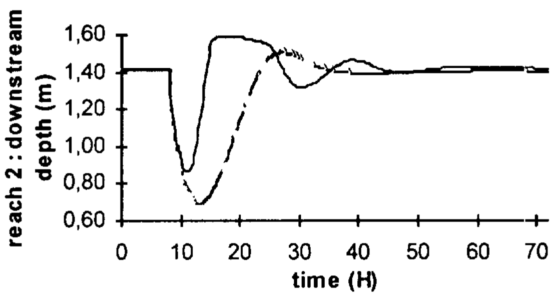
Biefs	Initial state	Variation imprévue de la demande à t= 8h
1	Upstream flow : 1.06 m^3/s	- 0,200
2	Upsteam flow : 0.886 m^3/s	- 0,200
3	Upstream flow : 0.870 m^3/s	- 0,200

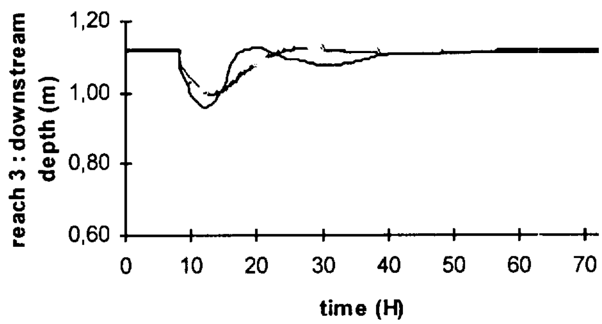












----- = monovariable controller
_____ = multivariable controller

For the design and implementing of a closed loop controller on a canal, automation and hydraulic engineers have the choice between :

- a monovariable controller which controls each reach independently,
- a multivariable controller which considers all the reaches as a single system.

The above simulation curves show satisfactory behaviour of both methods.

Multivariable controller has the advantage of better distributing the effects of a disturbance throughout the canal reaches. Through these objectives optimum control limits the variations of gates movement.

Monovariable controller with a Smith predictor type controller is very easy to implement. However, it requires coordination of adjustments which in Dynamic Regulation software are taken into account by the carry over of corrections from downstream to upstream.

CONCLUSION

This article describes two controllers which can be integrated in the Dynamic Regulation software closed loop module.

- A monovariable controller adapted to the delay systems controls the level downstream of a reach using upstream discharge. The synthesis of the controller is obtained using a pole placement technique. The use of this type of controller requires coordination of adjustments which can be made by carry over the corrections from downstream to upstream.
- A multivariable controller, the aim of which is to minimize one criterion. The multivariable representation of a reach is the space state of dimension 5, and this reduces the difficulty of numerical calculations.

For the simulation proposed, the behaviour of both controllers is satisfactory.

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