Predictive Control Applied to ASCE Canal 2

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ABSTRACT

Applying automatic control methods to irrigation canals is a way to improve the management of irrigation systems. The difficulties involved by these hydraulic systems such as non-linearity, delays and uncontrolled perturbations make the choice of a suitable automatic control method a challenge. To this purpose, this paper intends to investigate the applicability of control strategies which would be able to compensate the time delay and to overcome the system non-linearity. The proposed method is the so-called Generalized Predictive Control technique. After a brief overview of open canals flow modeling we recall the basic principles of the G.P.C. Then, we are interested in the application of this method to the three first reaches of the canal2 proposed by the ASCE Task Committee on Canal Automation Algorithms. In our study, the objective is to regulate the downstream water level of each reach by controlling both upstream discharges and thus gate openings.

1. INTRODUCTION

The main objectives of irrigation canals regulation are: improving water efficiency and distribution, reducing water losses and supplying water users in due time. Considering the difficulties and constraints involved by manual systems, research groups become more interested in developing new methods of automatic control for open canals [2], [10], [12]. Many studies have shown that classical regulators such as the Proportional - Integral - Derivative controller seem to be unsuitable to irrigation systems because of the important time delay characterizing these systems [3], [12]. In this study, we suppose that the canal can be represented as a succession of reaches separated by: cross-gates, weirs, etc... . The discharge and the water level are both mathematically modeled by the well known de Saint-Venant's equations [13]. The temporal variations of the water level and the discharge all along the canal are computed using a software which resolves the discretized equations of the previous model [1], [2].

The paper is organized as follows: in the second section a modeling of an irrigation canal is done by the mean of the de Saint-Venant's equations which describe the hydraulic phenomena in the canal. The third section exposes briefly the Generalized Predictive Control algorithm that is based on a minimization of a quadratic criterion so as to make the output follow the desired setpoint over a prediction horizon. The last part focuses on the application of the Generalized Predictive Control algorithm to downstream water level regulation in a portion - first three reaches - of canal2 proposed by the ASCE Task Committee on Canal Automation Algorithms. The control is accomplished by the means of the upstream discharges and the gate openings.

2. MODELING OF AN IRRIGATION CANAL

It is supposed that the canal can be described as a succession of reaches separated by hydraulic structures such as: cross-gates, weir ,..etc.. . Discharges can also be withdrawn at any time and generally at the downstream end of a reach. Hydraulic flows in the canal are described by the following de Saint-Venant's equations:

$$\partial S/\partial t + \partial Q/\partial x = q$$
$$\partial Q/\partial t + \partial(Q^2 / S)/\partial x + gS(\partial S/\partial x) = -gSJ + kqV$$

where: $Q(x,t)$: discharge, $Z(x,t)$: water level, S: wet section, J: linear discharge losses, V: mean velocity, q: lateral discharge, k: weighting coefficient, g: gravity acceleration, t: time variable, x: space variable.

This model is a distributed parameter equations system whose solutions are obtained after a discretization using the Preissman's scheme. This work has resulted in the design of a software able to calculate the discharge and the water level all along the canal at each sample during a given period [1], [2]. The curves obtained for different

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examples of reaches show that the canal length and the discharge delivered upstream influence directly the delay duration and variation.

3. THE GENERALIZED PREDICTIVE CONTROL THEORY

Due to the long time lag appearing in irrigation systems, the chosen regulation method is designed in order to be insensitive to the value and the variation of the delay. The principle is to anticipate the open canal time lag by predicting the output over a time horizon called: the prediction horizon. We use the Generalized Predictive Control horizon (G.P.C.) which is based on [7], [8]:

The process model
Any physical system can be represented by a locally linearized sampled model. In the Generalized Predictive Control method and for each reach, we use the well known A.R.I.M.A.X. model given by:

$$A(q^{-1})y(k) = B(q^{-1})u(k-d) + \frac{C(q^{-1})}{\Delta(q^{-1})}\xi(k)$$

where:

$$A(q^{-1}) = 1 + a_2 q^{-1} + \dots + a_{na+1} q^{-na}$$

$$B(q^{-1}) = b_1 + b_2 q^{-1} + \dots + b_{nb+1} q^{-nb}$$

$$C(q^{-1}) = 1$$

$$\Delta(q^{-1}) = 1 - q^{-1}$$

$y(k)$, $u(k)$, $\xi(k)$, and d are respectively the downstream water level variation, the upstream discharge variation, a sequence of random variables, and the time delay.

The optimal j-step ahead predictor
At the instant (k+j), the downstream discharge is given by:

$$y(k+j) = \frac{B(q^{-1})}{A(q^{-1})}u(k+j-d) + \frac{1}{A(q^{-1})\Delta(q^{-1})}\xi(k+j)$$

Using the following Diophantine equation :

$$1 = E_j(q^{-1})A(q^{-1})\Delta(q^{-1}) + F_j(q^{-1})q^{-j}$$

with:

$\deg E_j(q^{-1}) = j-1$ and $\deg F_j(q^{-1}) = \deg A(q^{-1})$.

$E_j(q^{-1})$ and $F_j(q^{-1})$ are uniquely defined by : $A(q^{-1})$ and j.

Thus the optimal j-step ahead predictor is:

$$\hat{y}(k+j/k) = G_j(q^{-1})\Delta u(k+j-d) + F_j(q^{-1})y(k)$$

where: $G_j(q^{-1}) = B(q^{-1})E_j(q^{-1})$

for $j \le d$, $\hat{y}(k+j/k)$ depends on available data but for $j > d$, $\hat{y}(k+j/k)$ depends on variables which have to be determined. Thus the j-step ahead predictor can be divided into two parts. The first one is depending on available input variables while the second part is composed of unknown inputs to be calculated.
An important assumption about the future control increments is made in the G.P.C. algorithm. In fact, it supposes that at the present time all the increments are null after a certain control horizon: $\Delta u(k+j) = 0$ for $j \ge N_u$

For j varying from N_i (initialization horizon) to N_p (prediction horizon) we obtain the following vectorial equation:

$$\hat{y} = G\tilde{u} + p$$

where:

$$\hat{y} = [\hat{y}(k+N_i/k), \dots, \hat{y}(k+N_p/k)]$$

$$\tilde{u} = [\Delta u(k), \dots, \Delta u(k+N_u-1)]$$

$$p = [p(k+N_i), \dots, p(k+N_p)]$$

The first term of the right section of the above equation forms the predictable part, while the second forms the unpredictable part given by:

$$p(k+j) = F_j y(k) + \sum_{i=j-d+1}^{n_{g}} g_{j,i+1}\Delta u(k+j-d-i)$$

The design of the predictive control law
Assuming that the future setpoint is known (desired water level: y^*), the aim of the control is to make the downstream water level follow the setpoint over a given time horizon. To this purpose we define the criterion:

$$J(N_i, N_p, N_u, \lambda) = \sum_{j=N_i}^{N_p} [\hat{y}(k+j/k) - y^*(k+j)]^2 + \sum_{j=1}^{N_u} \lambda(j)\Delta u(k+j-1)^2$$

Where:

N_i : initialization horizon

N_u : control horizon

N_p : prediction horizon

$\lambda(j)$: weighting coefficient

Then the minimization of the previous criterion allows to get the analytic optimal control expression:

$$\tilde{u} = (G^T G + \lambda I)^{-1}(y^* - p)$$

The receding horizon control strategy assumes that only the input: $u(k) = u(k-1) + \tilde{u}(1)$ is applied to the process. At the next sample, the whole procedure is repeated until the nullification of the error: $e(k) = y(k) - y^*(k)$.

4. THE GENERALIZED PREDICTIVE CONTROL OF A PORTION OF CANAL2 SET BY THE ASCE TASK COMMITTEE ON CANAL AUTOMATION

The ASCE Task Committee on Canal Automation Algorithms has set up real canal conditions simulations tests for two canals. The aim is to provide researchers with a benchmark that would allow performance comparison between different canal regulation methods. Each of the canals is composed of eight reaches separated by cross-gates, the main difference remains in the slope of each of them.

Two tests scemarios are considered for each canal: the tuned test where the control parameters are applied to the same canal system, and the untuned test where the tuned control parameters are applied to canal systems which have different Manning coefficient and different gate discharge coefficient.

In our study, we will focus on the regulation of the three first reaches of the Canal2 (flat canal). The control objective is to regulate the downstream level in each canal by modifying upstream discharges and thus the gate opening.

Their lengths are respectively 7km, 3km and 3km. The bottom width is of 7m, the bottom slope is of 0.0001, the Strickler is of 50, and the drop at each gate is about 0.2m.

The initial head discharge is $11 m^3/s$. We suppose that the initial offtake withdrawals are of $1 m^3/s$ at each reach. The offtakes are located 5m from the downstream end of each reach. Thus, a discharge of $8 m^3/s$ is maintained constant at the tail end.

Unlike the original Canal2 test, here we suppose that after 2 hours, the third offtake gate opening is increased by 0.0268 m. The system is assumed to be under tuned conditions.

Each reach is identified using the Recursive Least Square algorithm with a sampling period of 10 mn.

Reach 1	$A(q^{-1}) = 1 - 1.6112q^{-1} + 0.6099q^{-2}$ $B(q^{-1}) = 0.0110 + 0.0055q^{-1} - 0.0048q^{-2}$
Reach 2	$A(q^{-1}) = 1 - 1.3484q^{-1} + 0.3855q^{-2}$ $B(q^{-1}) = 0.0010 + 0.0038q^{-1}$
Reach 3	$A(q^{-1}) = 1 - 1.8882q^{-1} + 0.9213q^{-2}$ $B(q^{-1}) = 0.0109 - 0.0195q^{-1} + 0.0083q^{-2}$

Table 1. Reaches linear models

The delay in the three reaches is supposed to be of 10mn.

The design G.P.C. parameters N_i, N_p, N_u and λ are chosen as follows:

	N_i	N_p	N_u	λ
Reach 1	1	2	1	0.1
Reach 2	1	2	1	0.1
Reach 3	1	2	1	0.1

Table 2. Reaches design parameters

The resulting curves are depicted here after (see next page). Figures (a), (c), (e) represent the upstream discharge in the first, second and third reach respectively. Figures (b), (d), (f) show the evolution of the downstream water level in the first, second and third reach respectively. Finally, figures (i) and (j) represent the two gates openings during the regulation.

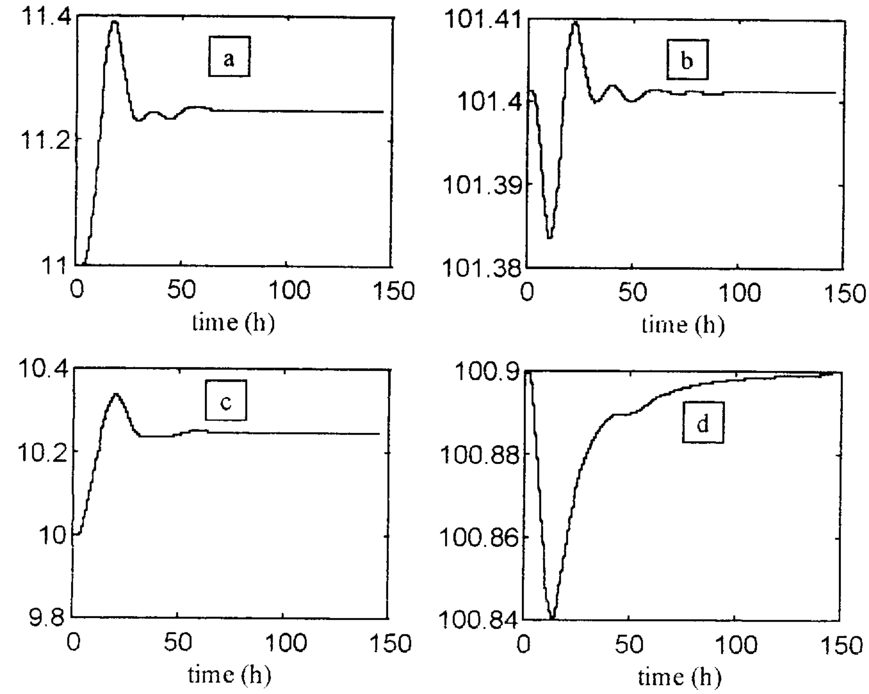
The performance of the regulators is also estimated, for the first twelve hours, using the performance indices defined by the ASCE Task Committee [11].

$$MAE_j = \frac{Max_{[t1...t2]}\left|y_j - y_{jtarget}\right|}{y_{jtarget}}$$

$$IAE_j = \frac{\Delta t}{(t_2 - t_1 + \Delta t) y_{jtarget}} \left| \sum_{t=t_1}^{t_2} (y_j - y_{jtarget}) \right|$$

$$StE_j = \frac{\Delta t}{(t_0 + \Delta t) y_{jtarget}} \left| \sum_{t=t_2-t_0}^{t_2} (y_j - y_{jtarget}) \right|$$

$$IAW_i = \sum_{t=t_1+\Delta t}^{t_2} \left|w_i(t) - w_i(t-\Delta t)\right| - \left|w_i(t_2) - w_i(t_1)\right|$$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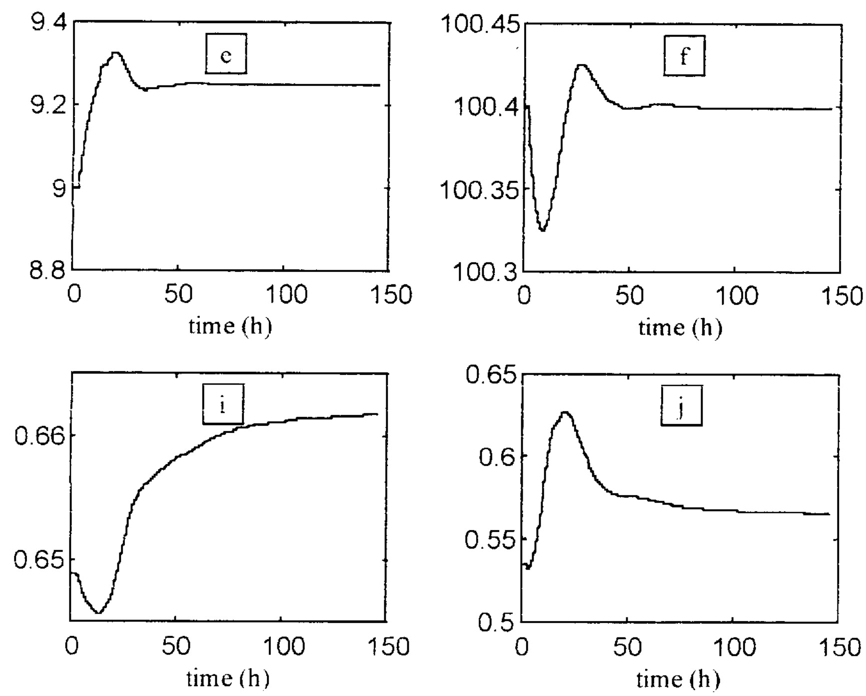
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Where: , $[t_1..t_2]$ = [[0h..12h], t_o=2 hours, y_j, $y_{jtarget}$, Δt, w_i and Q_i are respectively the downstream water level at each reach, the corresponding targeted value, the regulation ti:me step (10mn), the gate opening and the discharge at the gate.
The obtained indices are:

	MAE (%)	IAE (%)	StE (%)	IAW (%)	IAQ (%)
Period	0-12	0-12	0-12	0-12	0-12
Max.	23.85	1.17	6.99	0.55	0
Avg.	9.41	2.25	12.66	0.27	0

Table. 3 Performance indices

As the performance indices and the previous curves show, the change in the third offtake is compensated via the upstream discharges in each reach and the gates openings.

5. Conclusion

It stems from the following study that the Generalized Predictive Control is rather suitable to the regulation of a three-reach canal, as it allows to compensate a variation in an offtake discharge by slightly modifying upstream discharges of each reach and actuating gates between the considered reaches.

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