Multivariable predictive control of irrigation canals. Design and evaluation on a 2-pool model

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ABSTRACT
This paper presents the formulation of a multivariable controller for centralized control of canals based on the methodology of predictive control. The formulation relies on a linear state space model derived from Saint-Venant's equations discretized through the Preissmann implicit scheme. Predictive control uses this model to compute the control action that verifies a performance criterion defined over a finite prediction interval. The controller is combined with a Kalman filter to reconstruct the state variables and the unknown perturbations from a reduced number of measured variables, which are water levels at the upstream and downstream end of each pool. The control system is numerically tested on a 2-pool canal using a full non-linear simulation package (SIC).

INTRODUCTION
Control of canals is becoming increasingly considered as a tool for improving efficiency in water distribution for irrigation. Significant research efforts can be recognized in the literature [17]. Designing a control strategy leading to a practical controller is a complex task. Canals are systems distributed over long distances, with significant time delays and dynamics that change with the operating conditions. Check structures, like gates, in/outfakes, are placed along the canal at particular positions and strongly interact with the canal dynamics. Thus, the whole canal has to be regarded as a complex dynamic system with high number of variables related to states, outputs and inputs. One approach followed in the literature has consisted in seeing the canal as made of a number of subsystems or units and to deal with the problem of controlling each one of them in a monovariable setting. Different methods and controllers have been reported in the literature in which a single variable (flow, level or volume) is controlled by manipulating a single control variable (discharge or gate opening) [31], [10], [4]. One of the problems of this kind of approach is related to how to identify and deal with the interactions between the single units and the controllers [15].

Another approach is to consider the canal (or a significant part of it) as a multivariable system, with a number of state, output and input variables, aiming to design globally the control vector manipulating simultaneously a number of gates using the response of a number of sensors distributed along the canal. This kind of approach takes into account the coupling among subsystems but, because of the number of variables involved, is usually more complicated from a design and implementation point of view.

Multivariable controllers for canal automation have been designed based on optimal control techniques [7], [8], [3], [9], [24], [15], [12], non-linear optimization [11], and model inversion [14]. In this paper a multivariable controller is proposed based on predictive control techniques [22]. In previous works in the literature on canal automation, several authors have used predictive control: Rodellar et al. [25], [26], Sawadogo et al. [27], [28], [29], Compas et al. [6], Akouz et al. [1]. All these authors used a single or a series of monovariable predictive controllers in order to control a river or an irrigation canal. All of them used the simplified version of predictive control consisting of taking weighting coefficients only for the final states and considering a constant control action during the prediction interval. Some of them used the potential of the relatively simple mathematical formulation of the control parameters to design an adaptive controller that could follow model changes [26].

Using a framework like the one by Malaterre [15], this paper formulates a multivariable predictive controller based on a state model of the whole canal including a number of control gates, a number of states and a number of controlled and measured outputs. The controller is formulated in discrete time and combined with a Kalman filter to estimate states from a reduced number of measured outputs. It is named MPC for “Multivariable Predictive Control”. The complete control system is tested by performing numerical simulations of control operation under unknown perturbations on a 2-pool part of a canal set by the ASCE Task Committee on Canal Automation Algorithms [2].

LINEAR MODEL
To apply the MPC method, a linear model is required. The latter can be obtained analytically [15] or from transfer function identification [12]. The first option is selected in this paper. Saint Venant equations are discretized with Preissmann's implicit scheme, replacing the partial derivatives by finite differences. The canal is divided into a number of cross sections and two variables are considered at each section: the water elevation and the flow discharge. Appropriate boundary conditions need to be specified at outakes, check gates, head and tail end of the system. The control action variables are the upstream discharge and the check gate opening. The perturbation variables are the unknown offtake outflows. All variables are relative to the reference initial steady state.
In a discrete-time setting, the final model has the form:
\[
\begin{align*}
\dot{x}'(k+1) &= A'x'(k) + B'u(k) + B'p(k) \\
y(k) &= C'x'(k)
\end{align*}
\]
where \(x' \in \mathbb{R}^n\), \(u \in \mathbb{R}^m\), \(y \in \mathbb{R}^l\), \(p \in \mathbb{R}^p\), are respectively state, control action, controlled variables, and perturbation vectors at each time step \(k\). \(A', B', B'p', \) and \(C'\) are matrices of appropriate dimensions. The perturbation vector \(p\) is composed of a combination of discharge values at offtakes at the present and next time steps \(k\) and \(k+1\) [15].

In the case of tracking control, integral states \(x_I\) are appended to the system state vector \(x'\) in order to remove steady state errors at each controlled variable. Their dynamics is defined as:
\[
y_I(k+1) = x_I(k) + y(k+1) - y_{sp}(k+1)
\]
where \(y_{sp}\) denotes the setpoint. The system dynamics including the integrator can be written as:
\[
\begin{align*}
\dot{x}(k+1) &= Ax(k) + Bu(k) + e(k) \\
y(k) &= Cx(k)
\end{align*}
\]
where \(x(k) = [x', x_I]\).

**CONTROL DESIGN**

**Predictive controller**

The strategy underlying predictive control can be summarized as follow [22]: at each sampling instant \(k\), a model of the plant is used to predict the response over a finite time interval \([k, k+\lambda]\). Then, a control sequence is computed such that a performance criterion is verified over this interval. The control applied to the plant at time \(k\) is the first control belonging to this sequence.

In this work, the predictive model is based on the state equation (3) redefined at each time \(k\) with the actual state vector \(x(k)\). This can be written in the form:
\[
\begin{align*}
x(k+j|k) &= Ax(k+j-1|k) + Bu(k+j-1|k) + e(k+j-1|k) \\
x(k|k) &= x(k)
\end{align*}
\]
where \(x(k+j|k)\) denotes the state vector predicted at instant \(k\) for the future instant \(k+j\), \(u(k+j-1|k)\) denotes the control sequence and \(e(k+j-1|k)\) is the future perturbation sequence. Since this sequence has to be specified at instant \(k\), it can be either known (if perturbation are anticipated), estimated or approximated.

The performance criterion is selected with the linear quadratic form:
\[
J_k = \frac{1}{2} \sum_{j=1}^{\lambda} (x(k+j|k) - x_I(k+j|k))^T Q x(k+j|k) + \frac{1}{2} \sum_{j=0}^{\lambda-1} u(k+j|k)^T R u(k+j|k)
\]
where \(x_I(k+j|k)\) is a reference trajectory introduced to impose a desired tracking. \(Q\) and \(R\) are respectively a non-negative and a positive definite symmetric weighting matrix.

By applying (4) recursively we may write:
\[
\begin{align*}
x(k+1|k) &= Ax(k) + Bu(k|k) + e(k|k) \\
x(k+2|k) &= A^2x(k) + ABu(k|k) + Bu(k+1|k) + Ae(k|k) + e(k+1|k) \\
&\vdots \\
x(k+\lambda|k) &= A^\lambda x(k) + A^{\lambda-1}Bu(k|k) + A^{\lambda-2}Bu(k+1|k) + \ldots + Bu(k+\lambda-1|k) \\
&\quad + A^{\lambda-2}e(k|k) + A^{\lambda-3}e(k+1|k) + \ldots + e(k+\lambda-1|k)
\end{align*}
\]
Let:
\[
X = [x(k+1|k)^T, \ldots, x(k+\lambda|k)^T]^T, X' = [x'(k+1|k)^T, \ldots, x'(k+\lambda|k)^T]^T
\]
\[
U = [u(k)^T, \ldots, u(k+\lambda-1|k)^T]^T, E = [e(k)^T, \ldots, e(k+\lambda-1|k)^T]^T
\]
\[
Z = \begin{pmatrix} A & A^2 & \ldots \\ A & A^2 & \ldots \\ \vdots & \vdots & \ddots \end{pmatrix}, M = \begin{pmatrix} I & I & \ldots \\ \vdots & \vdots & \ddots \end{pmatrix}, V = \text{diag}(B, \ldots, B), Q = \text{diag}(Q_1, \ldots, Q_{\lambda}), R = \text{diag}(R_{u1}, \ldots, R_u)
\]
Then equation (6) can be written in the form:

\[ X = Z \cdot x(k) + M \cdot V \cdot U + M \cdot E \]  

(7)

and the performance criterion (5) as:

\[ J_k = \frac{1}{2} (X - X^*)^T Q (X - X^*) + \frac{1}{2} U^T R U \]  

(8)

Substitution of (7) into (8) easily leads to the minimization of \( J_k \) by the vector:

\[ U = -L Z \cdot x(k) + L \cdot x^* - L \cdot M \cdot E \]  

(9)

Where

\[ L = (R + (M \cdot V)^T \cdot Q \cdot M \cdot V)^{-1} (M \cdot V)^T \cdot Q \]

The control \( u(k) \) applied at instant \( k \) is made of the first \( m \) lines of vector \( U \).

**Discrete-Time Observer**

The control law (9) assumes that the complete system state vector \( x(k) = \begin{pmatrix} x' \\ x_I \end{pmatrix} \) can be measured accurately, which is often unrealistic. Most frequently, only certain linear combinations of states, denoted observed variables \( z(k) \), can be measured:

\[ z(k) = D' \cdot x'(k) \]  

(10)

where \( z \in \mathbb{R}^q \), and \( D' \) is a \((q, n)\) matrix. From variable \( z(k) \), the state vector \( x'(k) \) (and therefore \( x(k) \)) can be reconstructed. Then, the actual state \( x(k) \) is replaced by the reconstructed state \( \hat{x}(k) \) in (9). Due to unknown perturbations acting on the system, a state Kalman filter including a perturbation observer is designed.

The state Kalman filter is defined as:

\[ \hat{x}'(k+1) = A' \cdot \hat{x}'(k) + B' \cdot u(k) + B' \cdot \hat{p}(k) + N \cdot [z_k - D' \cdot \hat{x}'(k)] \]  

(11)

where \( \hat{p}(k) \) is the perturbation vector estimation:

\[ \hat{p}(k+1) = \hat{p}(k) + N \cdot [z_k - D' \cdot \hat{x}'(k)] \]  

(12)

\( N \) and \( N_p \) matrices can be computed by pole placement (Luenberger observer), or through the minimization of the reconstruction error (Kalman filter). The second option is tested in this paper. In steady conditions the global observer (11) plus (12) guarantees the vanishing of the reconstruction error and reconstructs accurately the perturbation acting on the system [15].

**Tuning**

Tuning parameters of the control algorithm are the weighting matrices \( Q_x \) and \( R_u \) of the performance criterion in (5) and the prediction horizon \( \lambda \), as well as the corresponding weighting matrices \( Q_o \) and \( R_o \) associated to the Kalman filter [15]. \( Q_x \) is defined as:

\[ Q_x = \begin{pmatrix} C' \cdot Q \\ C \\ 0 \end{pmatrix} \begin{pmatrix} Q_x \\ C \end{pmatrix} \]

which means that the controlled variable \( y \) is weighted with \( Q_y \) and the integral state vector \( x_I \) is weighted with \( Q_I \).

All these parameters have been tuned through a try and error procedure, leading to \( Q_x = I \), \( Q_y = 0.1 \cdot I \), \( R_u = [10 \ 200] \) and \( \lambda = 10 \) time steps. Since the sampling period is fixed to 5 minutes, \( \lambda = 10 \) means to consider a prediction interval of 50 minutes ahead. Another interesting tuning method using the Grammien matrices was also tested [13]. It directly gives the \( Q_x \) and \( R_u \) matrices from only one tuning parameter. It gave very good results, increasing the robustness of the controller, although slightly degrading the performance indicators. For the Kalman filter, the following weighting matrices have been chosen: \( Q_o = 100 \cdot I \) and \( R_o = 0.01 \cdot I \).

**Simulation Results and Analysis**

The process to be controlled is a 2-pool open-canals receiving water from a source located upstream. It is a portion of Canal 1 (Figure 1) of the test cases set by the ASCE Task Committee on Canal Automation Algorithms [2].

The control system aims to match the water level at the downstream end of each pool with a constant target value (2 controlled variables). Therefore, the reference state \( x(k+1|k) \) is chosen equal to 0. To reach this aim, the control system adjusts upstream inflow and opening of cross gate (2 control action variables). The observed variables, used by the Kalman filter, are water levels at the upstream and downstream ends of each pool (4 measured variables). No other variable is measured on the system, in particular no discharge is measured along the system and no information is measured at the offtakes.

The discrete-time linear model (1) is generated by a special module of the computer package SIC, developed by Cemagref [5], with a sampling interval of 5 minutes and a space step of 50 m (at the downstream section of the pools) and 500 m (at the upstream section of the pools). The dimension of the obtained state vector \( x' \) is 34.
The controller design is carried out with the commercial package MatLab [21]. The simulations, tests and computation of performance indicators are carried out on the full non-linear simulation model SIC, with a sampling interval of 5 minutes and a space step of 50 m.

At the check gates, a minimum gate movement of 0.5% of the gate height was suggested in the bench mark specifications [2]. As a simplification this constraint was not considered in this paper.

Performance indicators

In order to assess the performance of the controller, 3 indicators are computed for each controlled variable: the maximum absolute error (MAE), the integral of absolute magnitude of error (IAE), the steady state error (StE), and 2 indicators are computed for each control action variable, when this is relevant: the integrated average absolute gate movement (IAW) and the integrated average absolute discharge change (IAQ). These indicators are computed for 2 periods \([t_1, t_2]\): \([0h..12h]\) and \([12h..24h]\). These indicators are defined as

\[
\text{MAE}_j = \frac{\max_{t_1 .. t_2} | y_j(t) - y_j(t)_{\text{target}} |}{y_j(t)_{\text{target}}}
\]

\[
\text{IAE}_j = \frac{\Delta t}{(t_2 - t_1 + \Delta t)} \sum_{t = t_1}^{t_2} | y_j(t) - y_j(t)_{\text{target}} |
\]

\[
\text{StE}_j = \frac{\Delta t}{(t_2 - t_1 + \Delta t)} \sum_{t = t_1}^{t_2} | y_j(t) - y_j(t)_{\text{target}} |
\]

\[
\text{IAW}_i = \frac{b_2}{\Delta t} \sum_{t = t_1 + \Delta t}^{t_2} | w_i(t) - w_i(t - \Delta t) | - | w_i(t_2) - w_i(t_1) | \quad \text{for } i = 2
\]

\[
\text{IAQ}_i = \frac{b_2}{\Delta t} \sum_{t = t_1 + \Delta t}^{t_2} | Q_i(t) - Q_i(t - \Delta t) | - | Q_i(t_2) - Q_i(t_1) | \quad \text{for } i = 1 \text{ to } 2
\]

Where \(y_j, y_j(t)_{\text{target}}, \Delta t, w_i\) and \(Q_i\) are respectively the water elevation at the downstream end of pool \(j\) (controlled variable, \(j = 1 \text{ to } 2\)), the corresponding targeted value, the regulation time step (5 min.), the gate opening (control action variable for structure 2) and the discharge at structure \(i\) (control action variable for structure 1, resulting flow for structure 2).

Since this represents a large number of indicators, only the maximum and average values along the system are presented in the following sections.

Simulation results

The initial head discharge is 0.6 m3/s. The initial offtake withdrawals at offtake 1 and 2 are, respectively, 0.1 and 0.5 m3/s. The resulting tail end discharge is 0 m3/s.

After 2 hours, offtakes 1 and 2 increase their gate openings by 0.023 and 0.028 m, respectively, which should correspond, after steady state stabilization, to a discharge increase of 0.1 m3/s at both offtakes. Then, at time 14 h, offtakes 1 and 2 reduce their gate openings to get a 0.1 m3/s discharge decrease. The controller has no notice of these changes and can detect and correct their effects only through the measurement of the measured variables \(z\) (water levels at the upstream and downstream side of each check gate, at the head and at
This is called the “tuned conditions”. Figures 2 shows the evolution of the control action and controlled variables and of the flows at control structures as a function of time. The obtained performance indicators are presented in Table 1.

<table>
<thead>
<tr>
<th></th>
<th>MAE (%)</th>
<th>IAE (%)</th>
<th>Ste (%)</th>
<th>IAW (%)</th>
<th>IAQ (%)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Period 0-12</td>
<td>12-24</td>
<td>0-12</td>
<td>12-24</td>
<td>0-12</td>
<td>12-24</td>
</tr>
<tr>
<td>Max.</td>
<td>15.47</td>
<td>19.13</td>
<td>2.26</td>
<td>2.15</td>
<td>0.01</td>
</tr>
<tr>
<td>Avg.</td>
<td>14.1</td>
<td>15.3</td>
<td>1.77</td>
<td>1.68</td>
<td>0.01</td>
</tr>
</tbody>
</table>

Table 1. Performance indicators obtained in SIC for MPC (tuned conditions).

The same controller is tested without further tuning on the same system and same scenario except 3 modifications: Manning coefficient is 0.018 instead of 0.014, discharge coefficients at check gates are 10% lower (0.9 instead of 1.0), offtake withdrawals are 5% higher. This situation is referred to as “untuned conditions” and results are shown in Figure 3 and Table 2.

<table>
<thead>
<tr>
<th></th>
<th>MAE (%)</th>
<th>IAE (%)</th>
<th>Ste (%)</th>
<th>IAW (%)</th>
<th>IAQ (%)</th>
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</thead>
<tbody>
<tr>
<td>Period 0-12</td>
<td>12-24</td>
<td>0-12</td>
<td>12-24</td>
<td>0-12</td>
<td>12-24</td>
</tr>
<tr>
<td>Max.</td>
<td>16.75</td>
<td>21.46</td>
<td>2.5</td>
<td>2.35</td>
<td>0.02</td>
</tr>
<tr>
<td>Avg.</td>
<td>15.28</td>
<td>16.88</td>
<td>1.88</td>
<td>1.79</td>
<td>0.01</td>
</tr>
</tbody>
</table>

Table 2. Performance indicators obtained in SIC for MPC (untuned conditions).
All performance indicators are slightly degraded on the untuned conditions. But the results are still very good, illustrating robustness of the controller. This is also assessed with the ratio (presented in Table 3) of the performance indicators between the untuned (Table 2) and the tuned conditions (Table 1).

<table>
<thead>
<tr>
<th></th>
<th>ΔMAE (%)</th>
<th>ΔIAE (%)</th>
<th>ΔStE (%)</th>
<th>ΔIAW (%)</th>
<th>ΔIAQ (%)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Period</td>
<td></td>
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<td></td>
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<tr>
<td>0-12</td>
<td>8</td>
<td>12</td>
<td>11</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>12-24</td>
<td>9</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>92</td>
</tr>
<tr>
<td>Max.</td>
<td>8</td>
<td>12</td>
<td>11</td>
<td>9</td>
<td>92</td>
</tr>
<tr>
<td>Avg.</td>
<td>8</td>
<td>10</td>
<td>6</td>
<td>7</td>
<td>69</td>
</tr>
<tr>
<td>0-12</td>
<td>12-24</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Avg.</td>
<td>8</td>
<td>10</td>
<td>6</td>
<td>7</td>
<td>69</td>
</tr>
</tbody>
</table>

Table 3. Relative performance indicators obtained in SIC for MPC (untuned conditions compared with tuned conditions (in %)).

DISCUSSION

Results obtained in the simulations are satisfactory, as observed in the plots of Figures 2 and 3 and quantified in the performances indicators in Tables 1 and 2. Besides this general comment, some remarks can be drawn about the controller proposed in this paper.

One interesting issue is to discuss on the role of the length of the prediction interval $\lambda$. This is the main parameter to be tuned together with the weighting matrices in the performance criterion (5). Increasing $\lambda$ imposes to extend the prediction horizon, thus including longer period over which the performance criterion in (5) has to be minimized. Since the effect of the control actions requires time to travel further upstream and downstream, it seems expectable to notice an increase in the effect of the control actions on distant controlled water levels as the value of $\lambda$ increases. One way of assessing this increase has been considered in Figure 4. The control law (9) explicitly includes, in its closed loop part $LZx(k)$, a contribution proportional to the integrator vector $x_I(k)$. Since this vector is defined in terms of the output vector $y(k)$ in equation (2), the control law includes terms proportional to the controlled water levels $y_1$ and $y_2$ at the downstream end of each pool respectively. Thus the contribution of water levels $y_1(k)$ and $y_2(k)$ to the control $u_1(k)$ (upstream discharge) can be expressed in the form $\theta_1y_1(k) + \theta_2y_2(k)$, while the contribution to the control $u_2(k)$ (gate opening) is of the form $\theta_3y_1(k) + \theta_4y_2(k)$.
Figure 4 plots the values of \(|\frac{\theta_{11}}{\theta_{21}}|\) and \(|\frac{\theta_{12}}{\theta_{22}}|\) for increased values of \(\lambda\). We may observe as the effect of the control action \(u_1\) relative to \(u_2\) increases with \(\lambda\) for both the controlled levels, thus illustrating the increasing importance of the control action on distant controlled levels. This means that tuning the parameter \(\lambda\) allows to shift from a local to a distant control, which in this case corresponds also to a shift from an upstream to a downstream control. This shift has a limit indicated by the upper horizontal line in both plots of Figure 4, which have been obtained for the limit case when \(\lambda\) is made infinite. This case leads the approach of predictive control to the one derived using the asymptotic solution of the linear quadratic (LQ) optimal controller [15].

![Figure 4. Modification of the control direction ratio with \(\lambda\) for MPC and comparison with the LQ corresponding value.](image)

The selection of parameter \(\lambda\) supplies a degree of freedom for the use of predictive controller that can be useful in moving the control effort upstream or downstream as discussed above in face of using a LQ approach with infinite time. Increasing the value of \(\lambda\) increases the computational effort to calculate the gain matrix in the control law so that the approach presented in this paper may require more computation effort than the asymptotic solution of the Riccati equation in the LQ problem. Thus a compromise between this effort and the flexibility of the design can be required for using the controller. In this sense it is important to note that the approach presented here can be simplified by imposing a constant control action over the prediction interval and weighting only the final time step in the performance criterion (5). This will lead to significantly more simple formulations [22]. This simplification has not been tested in this paper and results will be available in the near future.

Comparison between the predictive controller proposed in this paper and a standard asymptotic LQ controller was made in a previous paper [20]. Results obtained by both controllers are close. Weighting matrices \(Q_y=I\), \(Q_z=I\) and \(R=1000(1,10)\) have been selected by a try and error procedure. Experience shows that tuning of these matrices is more difficult and results are more dependable than for the predictive controller, since this one relies much more on tuning the prediction horizon \(\lambda\) than the weighting matrices. On the other hand, the asymptotic LQ controller requires less computational effort, and appeared to be more robust.

**SUMMARY AND CONCLUSION**

This paper presents a method for automatic multivariable control of irrigation canals. It is numerically tested on a 2-pool canal. By adjusting the upstream discharge and the gate opening a target water level is maintained at the downstream end of each pool. Predictive control provides a state controller to perform this operation in conjunction with an observer that estimate the states from selected measured water levels. The controller is easily tunable mainly through a parameter that gives the system the possibility of shifting the control effect upstream or downstream the canal. The performance is satisfactory in terms of a quick evolution to target water levels and recovering of negative effects due to unknown disturbances such as unpredicted water withdrawals. Thus the control method seems well suited for real-time operations.

**REFERENCES**


