STABILITY AND PERFORMANCE ANALYSIS OF CLASSICAL DECENTRALIZED CONTROL OF IRRIGATION CANALS

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Abstract: Irrigation canals have a series structure which is generally used to design multivariable controllers based on the aggregation of decentralized monovariable controllers. SISO controllers are designed for each canal pool, assuming that the interactions will not destabilize the overall system. It is shown that, when the canal pools are controlled using the discharge at one boundary, the multivariable decentralized control structure is stable if and only if the SISO controllers are stable. The performance of the multivariable system is also investigated, and it is shown that the interactions decrease the overall performance of the controlled system. This loss of performance can be reduced by using a feedforward controller. Experimental results show the effectiveness of the method.

Keywords: Irrigation canal, Decentralised control, Application, Stability and Performance Analysis

1. INTRODUCTION

Irrigation canals are used to conduct the water from its source (a river, a dam) towards its users (pumping stations or individual farmers). Managing irrigation canals efficiently *i.e.* satisfying water users and at the same time minimizing the losses of water resource is an increasingly important issue. It is recognized that automatic control can improve the management of irrigation canals.

An irrigation canal is a multivariable system presenting strong interactions between subsystems. However, a wide number of applications and many publications use a decentralized technique to design controllers for irrigation canals (Weyer, 2002), (Seatzu, 2000), (Schuurmans, 1997), (Deltour and Sanfilippo, 1998), (Baume *et al.*, 1999), (Reddy *et al.*, 1992). In these cases, simple controllers are first designed for each canal pool, and are used together in order to control the overall system. Usually, the classical distant downstream control policy is chosen for each pool, where the downstream water level is controlled using the upstream discharge. This is a monovariable controller design problem, usually solved with simple PI controllers. Feedforward controllers are then added in order to reduce the interactions between each canal pool (Schuurmans, 1997), (Jreij, 1997). Such a design method usually gives a correct controller for the whole system, since the system appears to be stable. However, this method has never been analyzed using modern automatic control tools (Skogestad and Postlethwaite, 1998).

Why do essentially monovariable techniques work on a multivariable system? In this paper the analysis of the question is done using theory and classical control tools. A systematic and rigorous methodology for analysis and design of linear decentralized controllers for a canal with multiple pools taking into account the interactions between each pool is proposed. It is explained why the decentralized control method leads to a stable multivariable closed-loop system. The robustness and the performance of the closed-loop are also investigated. It is shown that the static feedforward

controller classically used by hydraulic engineers can be improved by using a dynamic controller.

2. PROBLEM FORMULATION

An irrigation canal can be represented as a series of pools (see figure 1). Each pool represents a portion of canal in between two hydraulic structures (gates or weirs for example). For the ith pool, we denote u_i the control variable (discharge) at the upstream end, u_{i+1} the control variable at the downstream end, y_i the controlled variable (water depth at the downstream of the pool i) and d_i the load disturbances (water offtake).

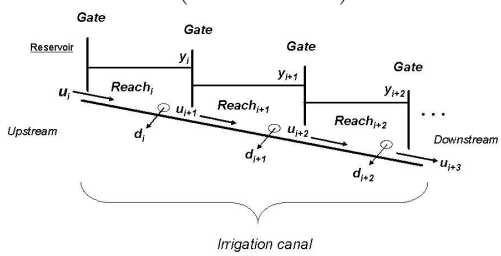

Fig. 1. Schematic longitudinal view of an irrigation canal

2.1 Modelling of the canal

The dynamics of each canal pool can be modelled by the so-called Saint-Venant equations, which are hyperbolic non-linear partial differential equations involving the discharge $Q(x,t)$ and the water depth $Y(x,t)$ along one space dimension (Chow, 1988). The hydraulic structures separating each pool are modelled by static nonlinear equations. We consider in the following a linear model of the canal, based on linearized Saint-Venant equations and linearized hydraulic structures equations.

2.1.1. Linear model based on the Saint-Venant equations The linearized Saint-Venant equations are used to obtain a transfer matrix representation of the system in the Laplace domain (as in (Litrico and Fromion, 2002)). The canal pool is then represented by:

$$y_i(s) = G_i(s)u_i(s) + \tilde{G}_i(s)(u_{i+1}(s) + d_i(s))$$

where the disturbance $d_i(s)$ (corresponding to the unknown withdrawal) is supposed to act additively with the downstream discharge. Litrico and Fromion have shown in (Litrico and Fromion, 2002) that the transfer functions have the following inner-outer factorization:

$$G_i(s) = G_{io}(s)e^{-\tau_i s}$$
$$\tilde{G}_i(s) = \tilde{G}_{io}(s)$$

with τ_i the time-delay for downstream propagation and where $G_{io}(s)$ and $\tilde{G}_{io}(s)$ are 'outer'. The delay τ_i is obtained by $\tau_i = \int_0^{X_i} \frac{dx}{C_0+V_0}$, where X_i is the length of pool i. $C_0 = \sqrt{\frac{gA_0}{T_0}}$ is the celerity (m/s), with T_0 the top width (m), A_0 the wetted area (m^2), g the gravitational acceleration (m/s^2) and $V_0 = \frac{Q_0}{A_0}$ is the velocity (m/s) with Q_0 the discharge (m^3/s) across section A_0. (The index $_0$ is related to all terms in equilibrium regime).

2.1.2. Hydraulic structures We assume in the following that irrigation canals are controlled with the discharge at the boundary of each canal pool. However, in practice, canals are controlled using hydraulic structures (gates, weirs), represented by static nonlinear equations. The control structure therefore assumes that there is a slave controller on each hydraulic structure that is used to deliver the required discharge.

2.2 Distant downstream control policy for an irrigation canal

Many control policies have been proposed for the control of irrigation canals. A classification of canal control algorithms was made in (Malaterre *et al.*, 1998). We will present a classical control policy: the distant downstream control, which enables to have a parsimonious water management. Then we will study and analyze the robustness and performance of the closed-loop system in this case.

Distant downstream regulation of a canal pool consists in controlling the downstream water level y_i using the upstream control variable u_i (see figure 2). A schematic representation of the system

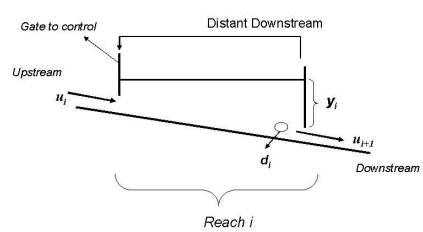

Fig. 2. Distant downstream control of one pool

for control purposes is depicted in figure 3. r_i is the reference signal, e_i the tracking error, and K_i the transfer function of the controller.

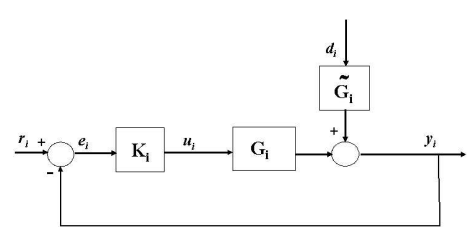

Fig. 3. Closed-loop system: Distant Downstream control

The sensitivity transfer function can be expressed by: $S_i = (1+G_iK_i)^{-1}$. The disturbances rejection is directly characterized by the modulus of the transfer function $-\tilde{G}_iS_i$. The control objective is to find a linear controller K_i such that

$$|\tilde{G}_i(j\omega)S_i(j\omega)| = \left|\frac{\tilde{G}_i(j\omega)}{(1+G_i(j\omega)K_i(j\omega))}\right| \approx 0$$

over the largest frequency bandwidth.

3. ANALYSIS OF MULTIPLE POOLS CANAL SYSTEM WITH DECENTRALIZED CONTROL

As already stated in introduction, a wide number of applications use a decentralized method to design controllers for multiple pools irrigation canals. It is therefore of great interest to analyze these classical methods with automatic control tools. We will study in this section the robustness and the performance of a multiple pools canal through the maximum singular value of the plant transfer matrix. Since disturbances are not known and are independent, the maximum singular value provides a good way to estimate the robustness and performance of the control system.

3.1 Distant downstream decentralized control of two pools system

For simplicity, but with no loss of generality, let us assume that the irrigation canal is composed of two pools in series, *i.e.* two SISO subsystems (see figure 4). It is simple to conclude that pool 1 will be affected by the control variable u_2, which acts as a disturbance on pool 1 because of the interaction between pool 1 and pool 2. When disturbance occurs in pool 2 the control variable u_2 acts on the gate situated on the upstream end of pool 2 in order to compensate the disturbance d_2. This produces a disturbance in pool 1 and the control variable u_1 acts on the gate situated on the upstream end of pool 1.

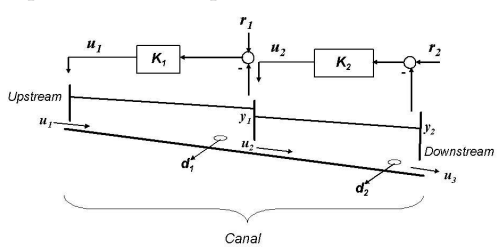

Fig. 4. Decentralized Distant Downstream control of two pool system

3.1.1. Stability and robustness analysis Taking into account the interactions between the SISO subsystems, the tracking errors e_1 and e_2 of the MIMO system are as follows:

$$e_1 = S_1 r_1 - \tilde{G}_1 S_1 d_1 + \tilde{G}_1 S_1 K_2 \tilde{G}_2 S_2 d_2$$
$$e_2 = S_2 r_2 - \tilde{G}_2 S_2 d_2$$

Denoting: $M_1 = -\tilde{G}_1 S_1$ and $M_2 = -\tilde{G}_2 S_2$ the relation between tracking errors e_1, e_2 and disturbances d_1, d_2 can be expressed as follows:

$$\begin{pmatrix} e_1 \\ e_2 \end{pmatrix} = \begin{pmatrix} M_1 & M_1 K_2 M_2 \\ 0 & M_2 \end{pmatrix} \begin{pmatrix} d_1 \\ d_2 \end{pmatrix} \qquad (1)$$

Since the transfer matrix M is upper triangular, the decentralized multivariable system naturally inherits the stability and robustness properties of the monovariable systems. Therefore, the multivariable system is stable *if and only if* all monovariable systems are stable. The monovariable input margins are also recovered in the multivariable case (Skogestad and Postlethwaite, 1998). This explains why the traditional decentralized control structure associated to distant downstream control with control inputs using discharge at the boundary works for irrigation canals.

3.1.2. Performance analysis We now investigate the performance of the multivariable system, by using the maximum singular value. One has:

$$\|e(j\omega)\| = \|M(j\omega)d(j\omega)\| \leq \bar{\sigma}(M(j\omega))\|d(j\omega)\| \qquad (2)$$

where $\bar{\sigma}(M(j\omega))$ is the maximum singular value of the transfer matrix $M(j\omega)$.

By definition of the maximum singular value, we know that there exists a couple $(\bar{d}_1(j\omega), \bar{d}_2(j\omega))$ such that the inequality (2) becomes an equality (the worst case perturbation). Since in the case of irrigation canals the perturbations are unknown, the maximum singular value is a good estimate of decoupling properties of the controlled canal.

Let us now characterize for a given frequency ω the maximum singular value of $M(j\omega)$, which is the square root of the maximum eigenvalue of $M(j\omega)M(j\omega)^*$, denoted

$$\mathcal{M}(j\omega) = \begin{pmatrix} |M_1|^2 + |M_1 K_2 M_2|^2 & M_1 K_2 |M_2|^2 \\ |M_2|^2 K_2^* M_1^* & |M_2|^2 \end{pmatrix}$$

where all transfer matrices are evaluated at $j\omega$. M^* denotes the transpose conjugate of M. In order that the interconnection does not degrade the performance, one would require:

$$\bar{\sigma}(M(j\omega)) \leq \max(|M_1(j\omega)|, |M_2(j\omega)|) \qquad (3)$$

Let λ_1 and λ_2 be the eigenvalues of the matrix $\mathcal{M}(j\omega)$. Their product is independent of the coupling: $\lambda_1\lambda_2 = |M_1|^2|M_2|^2$ and their sum $\lambda_1 + \lambda_2 = |M_1|^2 + |M_2|^2 + |M_1|^2|K_2|^2|M_2|^2$ is always greater than $|M_1(j\omega)|^2 + |M_2(j\omega)|^2$. Therefore, one has necessarily:

$$\bar{\sigma}(M(j\omega)) > \max(|M_1(j\omega)|, |M_2(j\omega)|)$$

which is in contradiction with inequality (3). Thus, the interaction between coupled subsystems necessarily decreases the performance of the overall multivariable system. A possible way to reduce this interaction is to use a feedforward controller.

3.1.3. Feedforward controller The feedforward controller generally used in the case of irrigation

canals has the following structure, depicted in figure 5. The "perturbation" generated by the interaction of the second pool on the first pool perfectly known, since it corresponds to the control input u_2, can be compensated by adding a feedforward term to the control structure. In this

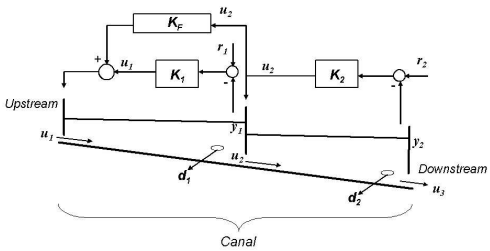

Fig. 5. Decentralized Distant Downstream control of two pools system with feedforward controller

case, the transfer matrix M becomes:

$$M_F = \begin{pmatrix} M_1 & M_1 K_2 M_2 \left(1 + K_F \dfrac{G_1}{\tilde{G}_1}\right) \\ 0 & M_2 \end{pmatrix}$$

The interactions between pools are expressed here by the non-zero offdiagonal element in matrix M_F. The perfect decoupling objective is then achieved if: $K_F = -\frac{\tilde{G}_1}{G_1}$. Assuming that transfers $\tilde{G}_1$ and G_1 are given respectively by: $\tilde{G}_1(s) = -\frac{1}{A_{d_1} s}$ and $G_1(s) = \frac{1}{A_{d_1} s} e^{-\tau_1 s}$, then $K_F(s) = e^{\tau_1 s}$ is a predictor, which is non causal.

To restate this problem in a rigorous way, the decoupling objective is obtained by minimizing the coupling term in matrix $M_F(j\omega) M_F(j\omega)^*$, denoted:

$$\mathcal{M}_F(j\omega) = \begin{pmatrix} |M_1|^2 + |M_1 K_2 M_2 Q|^2 & M_1 K_2 Q |M_2|^2 \\ |M_2|^2 Q^* K_2^* M_1^* & |M_2|^2 \end{pmatrix}$$

with $Q(j\omega) = 1 + K_F(j\omega) \frac{G_1(j\omega)}{\tilde{G}_1(j\omega)}$ the coupling term. We now search a feedforward controller $K_F^d(j\omega)$ such that the term $Q(j\omega)$ is minimal. This can be expressed as a H_∞ minimization problem: $\alpha = \inf_{K_F(s) \in H_\infty} \left\| 1 + K_F(s) \frac{G_1(s)}{\tilde{G}_1(s)} \right\|_\infty$. However, this does not lead to an interesting result, since Tannenbaum in (Tannenbaum, 1992) proved that: $\inf_{K_F(s) \in H_\infty} \|1 - K_F(s) e^{-\tau_1 s}\|_\infty = 1$. Therefore, it is not useful to minimize this norm over *all frequencies*, but it is necessary to specify a given frequency range where the decoupling should occur. In this case, a *weighted* H_∞ norm has to be considered.

In practice, hydraulic engineers use a constant gain (between 0.5 and 1). In this case, a gain of 1 should be chosen, since if $K_F(j\omega) = 1$, then $\|1 - e^{-\tau_1 j\omega}\| \approx 0$ for $\omega \approx 0$. However, there exists frequencies where $e^{-\tau_1 j\omega} \approx -1$ which lead to $|1 - K_F(s) e^{-\tau_1 j\omega}| \approx 2$. This is why a lead lag filter approximating $e^{\tau_1 s}$ over a given frequency range could lead to a better decoupling than a constant gain.

Remark: [Impact of delay uncertainty] Let us examine how a feedforward controller behaves when trying to compensate the effect of a sinusoidal perturbation known in advance $p_1(t) = a \sin(\omega_p t)$ at the downstream end of the pool. Then, the feedforward controller leads to $K_F(j\omega_p) = e^{\tau_1 \omega_p}$, which means that the control input should be the perturbation with a phase lag equal to $\phi = \tau \omega_p$. If the delay is not known precisely, but $\tau \in [\tau_1 - \Delta\tau, \tau_1 + \Delta\tau]$, then the possible phase lag due to this uncertainty is given by $\Delta\phi \in [-\Delta\tau\omega_p, \Delta\tau\omega_p]$ and for all frequencies such that $\omega > \omega_p^*$, with $\omega_p^* = \frac{\pi}{\Delta\tau}$, the sign of the feedforward control input is not known. In this case, a feedforward control could end up with doubling the perturbation rather than compensating it. A low pass filter should be added to the feedforward to eliminate such problem.

4. EXPERIMENTAL VALIDATION

After this theoretical analysis we test and validate the decentralized multivariable PI control in real conditions in the case of a distant downstream control policy.

4.1 Canal description

The canal used in the present study is a component of the experimental facility of the Hydraulics and Canal Control Center (NuHCC) of the University of Évora (Portugal). The experimental canal is a trapezoidal and lined canal, with a general cross section of bottom width 0.15 m, sides slope 1:0.15 and depth 0.90 m. The overall canal is 145.5 m long and the average longitudinal bottom slope is about 1.5×10^{-3}. The design flow is 0.09 $m^3 s^{-1}$. There is an offtake d_i at the downstream end of each pool. We consider a two pools system (see figure 4). The canal inlet is equipped with a motorized flow control valve, that delivers a discharge u_1 with the use of a local slave controller. An intermediate rectangular sluice gate, opening u_2, is used to control the inflow into reach n. 2.

4.1.1. Integrator Delay Zero (IDZ) model In order to design linear controllers and use classical tuning techniques we propose an analytical model of the system. A simplified model can be obtained following (Litrico and Fromion, 2004), by making suitable simplifications about the backwater curve. This leads to an Integrator Delay Zero (IDZ) approximation of transfer function $G_i(s)$ and an Integrator Zero approximation of transfer function $\tilde{G}_i(s)$, leading to the frequency domain model:

$$y_1 = \left(\frac{1}{19.8s} + 1.64\right) e^{-31.8s} u_2 - \left(\frac{1}{19.8s} + 1.86\right) u_3$$
$$y_2 = \left(\frac{1}{18.7s} + 1.68\right) e^{-35.5s} u_2 - \left(\frac{1}{18.7s} + 1.88\right) u_3$$

In order to show the accuracy of this approximate model we compare the 'complete' $G_i(s)$ and $\tilde{G}_i(s)$

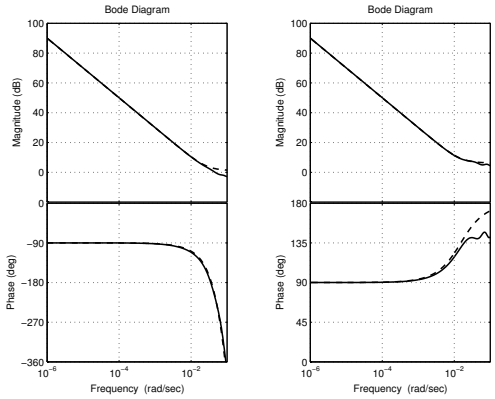

Fig. 6. Comparison: 'complete' $G_i(s)$ (left), $\tilde{G}_i(s)$ (right) transfers (-) and 'approximated' (IDZ) transfers (-.-)

transfers with the approximated transfers for the whole canal. As can be seen in figures 6 the approximate model fits very well the 'complete' model.

4.2 Controller design

Based on the previous analysis, we propose in the sequel a rigorous methodology to design a multivariable decentralized controller for an irrigation canal. First SISO distant downstream filtered PI controllers are tuned for each reach of the Évora canal. Then a robustness and stability analysis is done for each SISO system. We evaluate the relative performances of the control system with and without feedforward controller.

4.2.1. Linear controllers PI controllers are widely used in industry owing to their simplicity and robustness. We design filtered PI controllers for each reach in order to meet gain and phase margin specifications and to reject the load disturbance (offtake). The PI controllers have the following transfer function:

$$K_1(s) = 0.31\left(1 + \frac{1}{256s}\right)$$
$$K_2(s) = 0.26\left(1 + \frac{1}{45.45s}\right)$$

Controller parameters are tuned using a classical method proposed by Skogestad in (Skogestad, 2003).

4.2.2. Robustness analysis The use of a model-based method enables to easily evaluate the robustness of the control scheme by considering a family of linear models (in our case corresponding to different discharges). This analysis showed that the closed-loop system is stable for discharges varying from 10 to 80 l/s, which is a wide variation around the reference discharge (45 l/s).

4.2.3. Controller implementation The second pool of the canal is controlled with a rectangular sluice gate. Therefore, a method is needed to convert the computed discharge u_2 into a gate opening. Many different possibilities have been proposed in the literature (Malaterre and Baume, 1999). We tested experimentally various methods. Finally, we selected the simplest one, *i.e.* a local linear inversion of the hydraulic structure law.

4.3 Downstream PI robust control of two pools

In order to validate the proposed methodology, we compare experimental results with linear simulation (done in Matlab with the simplified IDZ model).

4.3.1. Without feedforward controller Figure 7 gives the experimental results obtained with the decentralized distant downstream controller. A

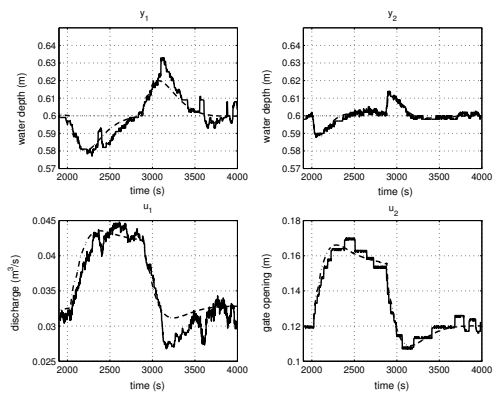

Fig. 7. Distant downstream control of two pools in series: experimental results (-) and linear simulation (-.-)

downstream withdrawal d_2 of 10 l/s is done at time $t = 2000$ s and stopped at time $t = 2800$ s. The decentralized controller reacts as expected: first the gate at the upstream end of pool 2 opens gradually in order to compensate the withdrawal occurred at the downstream end and bring back the output y_2 at the target $y_c = 0.6$ m. This opening produces a disturbance at the downstream end of the pool 1 and the water depth y_1 has a similar variation as water depth y_2. The discharge (input u_1) increases in order to compensate for the withdrawal.

4.3.2. With static feedforward controller The same experiment is repeated using a static feedforward. In figure 8 the downstream withdrawal d_2 of 10 l/s is done at time $t = 300$ s and stopped at time $t = 750$ s. The decentralised controller reacts correctly as in the case without feedforward controller. The essential difference can be observed for the variations of the water depth y_1. The output y_1 is much less sensitive to the disturbance d_2 that occurred at the the downstream end of pool 2. The water level y_2 is much closer to the target level $y_c = 0.6$ m. Thus, the feedforward controller improve the disturbance rejection. In both cases the linear simulations reproduce rather accurately the dynamic behavior of the closed-loop systems. This is remarkable, since the process includes

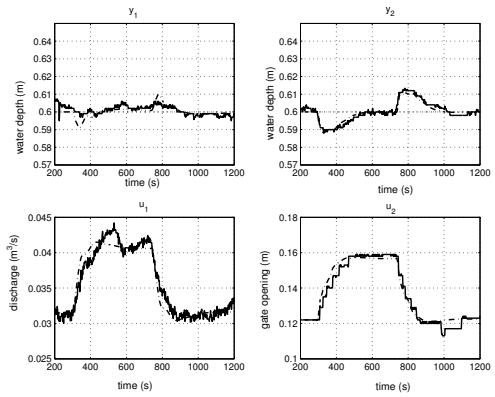

Fig. 8. Distant downstream control of two pools in series using a feedforward controller: experimental results (-) and linear simulation (-.-)

actuators nonlinearities and measurement noise. This validates our approach, from the modelling part (which is based on Saint-Venant equations) to the decentralized control method, that gives a satisfactory answer to the problem of irrigation canal management.

5. CONCLUSION AND FUTURE WORK

The paper has provided a detailed analysis in terms of stability, robustness and performance of decentralized controllers for irrigation canals. We have shown that, considering that the canal pools are controlled using the discharge at the boundary, *i)* the multivariable decentralized control structure is stable if and only if the SISO controllers designed for each canal pool are stable, *ii)* the robustness properties of the monovariable control systems are recovered by the multivariable control system for structured diagonal input uncertainties, *iii)* the interactions decrease the overall performance of the controlled system, *iv)* this loss of performance can be minimized by using a feedforward controller.

These results apply for distant downstream control structures and are experimentally validated on a real canal located in Portugal.

Future works will consider the stability analysis of mixed control policies, where in one pool both local upstream and distant downstream control are used.

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REFERENCES

Baume, J.P., P.O. Malaterre and J. Sau (1999). Tuning of PI controllers for an irrigation canal using optimization tools. In: *Proceedings of USCID Workshop on Modernization of irrigation water delivery systems*. Phoenix, Arizona, USA. pp. 483–500.

Chow, V.T. (1988). *Open-Channel Hydraulics*. McGraw-Hill Book Company.

Deltour, J.-L. and F. Sanfilippo (1998). Introduction of Smith predictor into Dynamic Regulation. *Journal of Irrigation and Drainage Engineering* **124**(1), 47–52.

Jreij, H. (1997). Sur la régulation des cours d'eau aménagés. PhD thesis. Université Paris - XI Dauphine.

Litrico, X. and V. Fromion (2002). Infinite dimensional modeling of open-channel hydraulic systems for control purpose. In: *Conference on Decision and Control CDC'02*. Las Vegas, Nevada, USA.

Litrico, X. and V. Fromion (2004). Analytical approximation of open-channel flow for controller design. *Applied Mathematical Modelling* **28**(7), 677–695.

Malaterre, P.-O. and J.P. Baume (1999). Optimum choice of control action variables and linked algorithms: Comparison of different alternatives. In: *Proceedings of USCID Workshop on Modernization of irrigation water delivery systems*. Phoenix, Arizona, USA. pp. 387–405.

Malaterre, P.O., D.C. Rogers and J. Schuurmans (1998). Classification of canal control algorithms. *Journal of Irrigation and Drainage Engineering* **124**(1), 3–10.

Reddy, J. M., A. Dia and A. Oussou (1992). Design of control algorithm for operation of irrigation canals. *Journal of Irrigation and Drainage Engineering* **118**(6), 853–867.

Schuurmans, J. (1997). Control of water levels in open-channels. PhD thesis. Delf University of Technology.

Seatzu, C. (2000). Decentralized controllers design for open-channel hydraulic systems via eigenstructure assignment. *Applied Mathematical Modelling* **24**(12), 915–930.

Skogestad, S. (2003). Simple analytic rules for model reduction and PID controller tuning. *Journal of Process Control* **13**(4), 291–309.

Skogestad, S. and I. Postlethwaite (1998). *Multivariable Feedback Control.Analysis and Design*. Wiley.

Tannenbaum, A. (1992). Frequency domain methods for the H_∞ optimization of distributed systems. Vol. 185. Springer-Verlag. New York.

Weyer, E. (2002). Decentralized PI Control of an Open Water Channel. In: *IFAC, 15th Triennial World Congress*. Barcelona, Spain.